

Introduction to the constrained equilibrium method

M. to Baben¹, P. Koukkari², R. Pajarre², G. Eriksson¹, K. Hack¹

1: GTT-Technologies

2: VTT Technical Research Centre of Finland Ltd



Motivation

- Most technical processes are not in equilibrium.
- Kinetic limitations impose constraints on the equilibrium.
- Simple methods can sometimes be very effective in describing constrained equilibria.



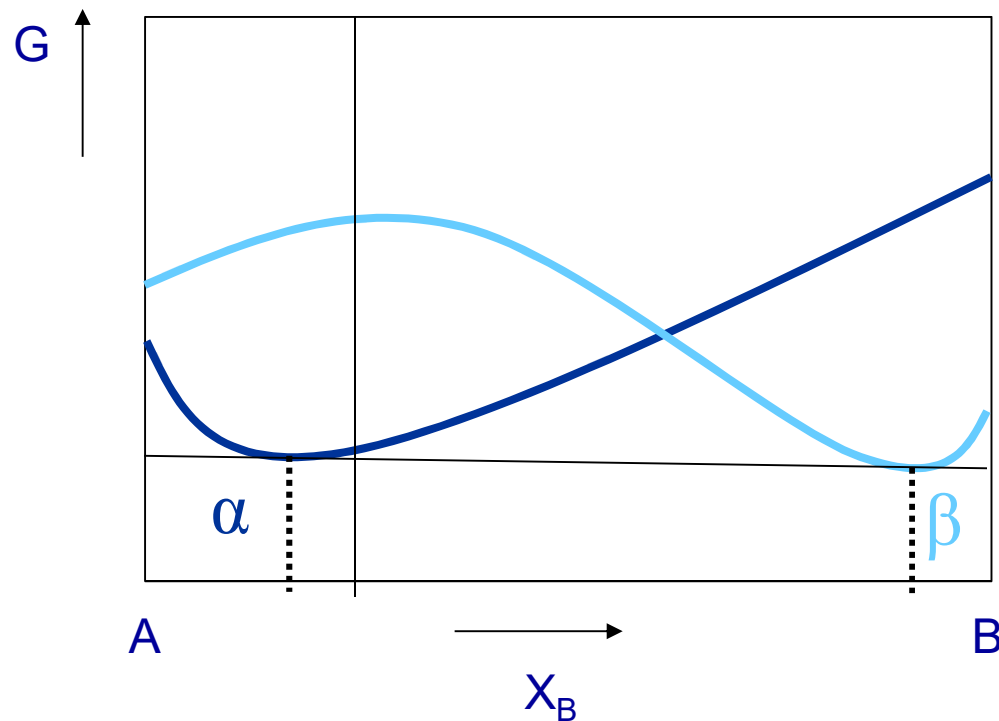
Outline

- Dormant species
- Paraequilibrium calculations with and without diffusing elements
- Scheil cooling calculations
- Immaterial system components (for work terms and reaction rate constraints)



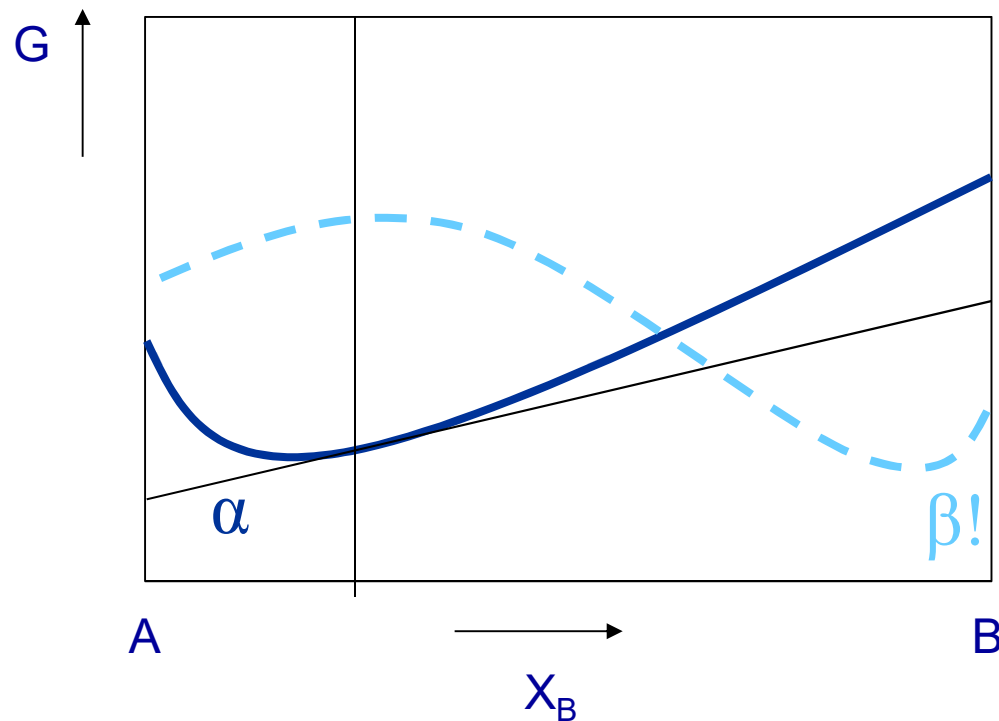
Dormant species

- Dormant species may not form, but their thermodynamic properties are calculated.



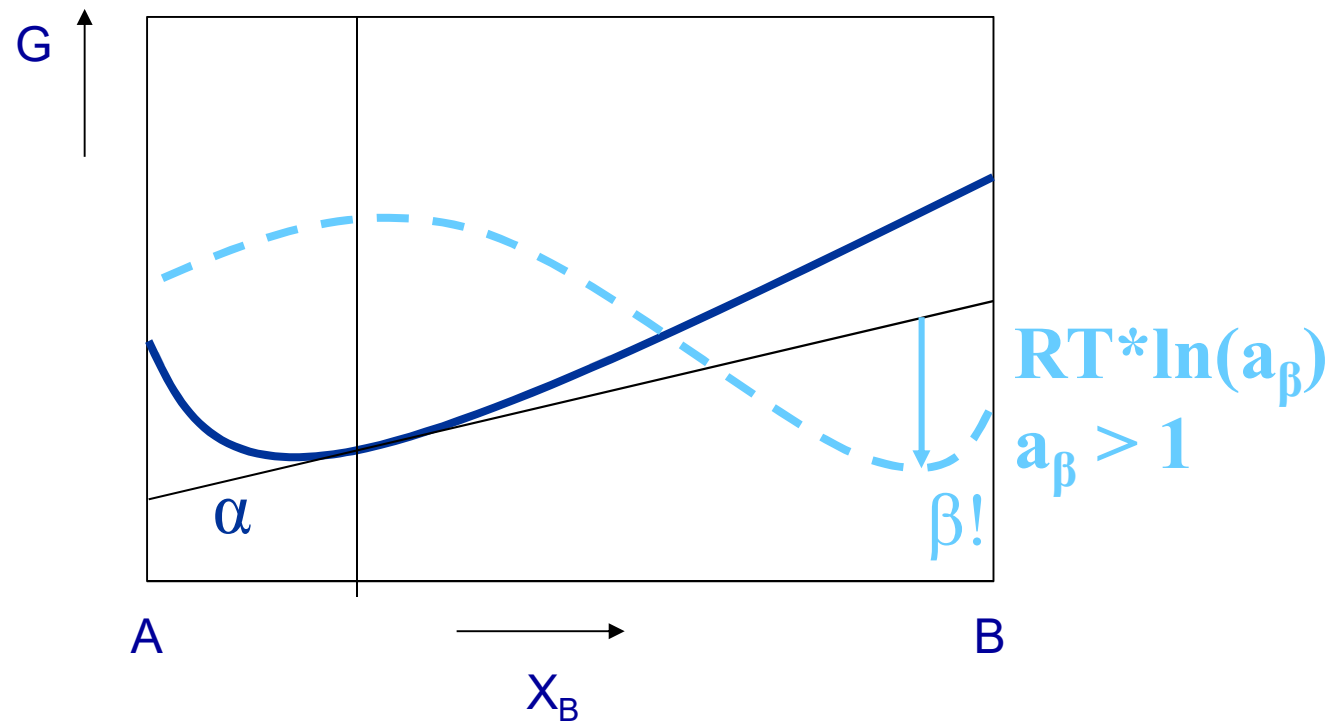
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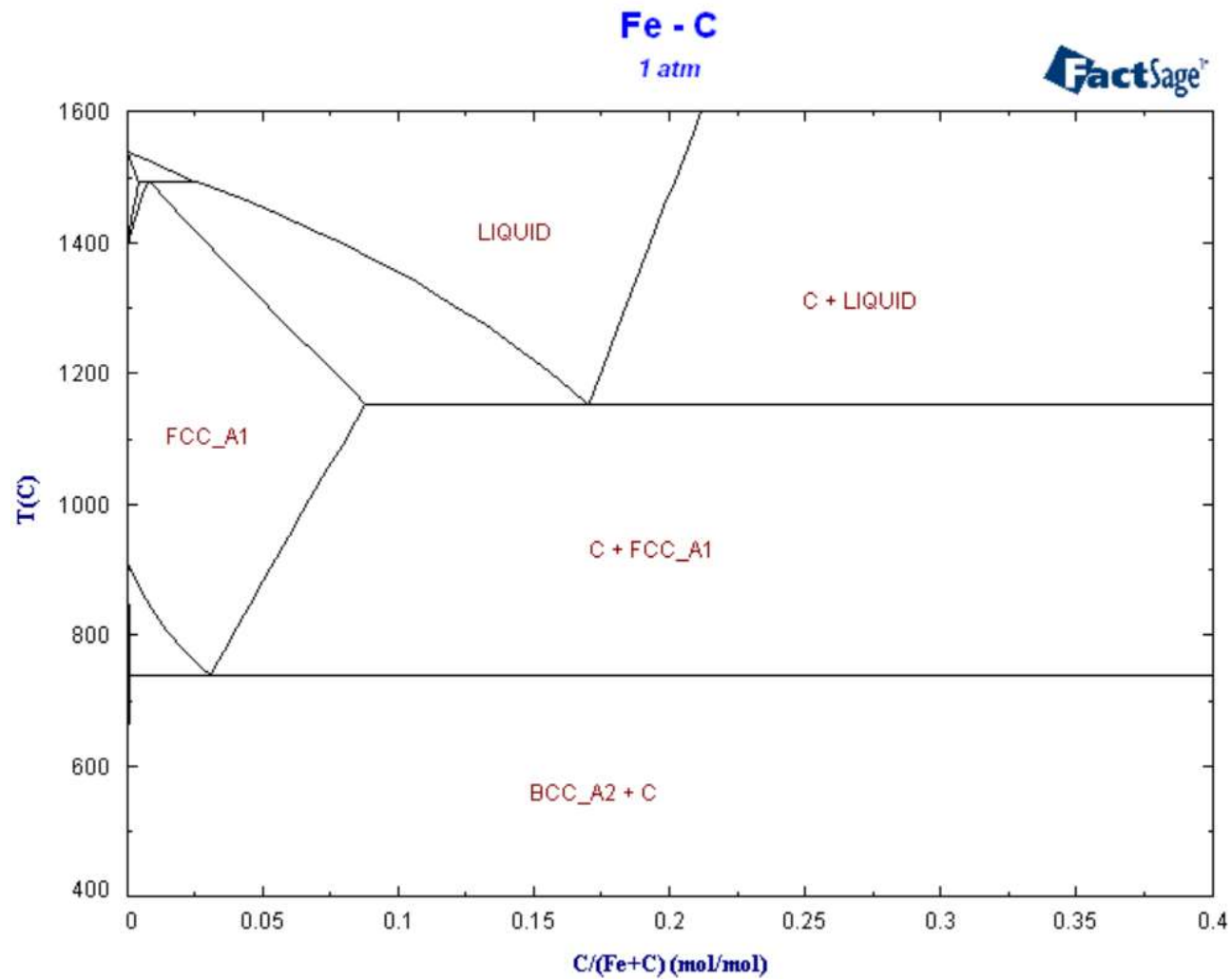


Dormant species

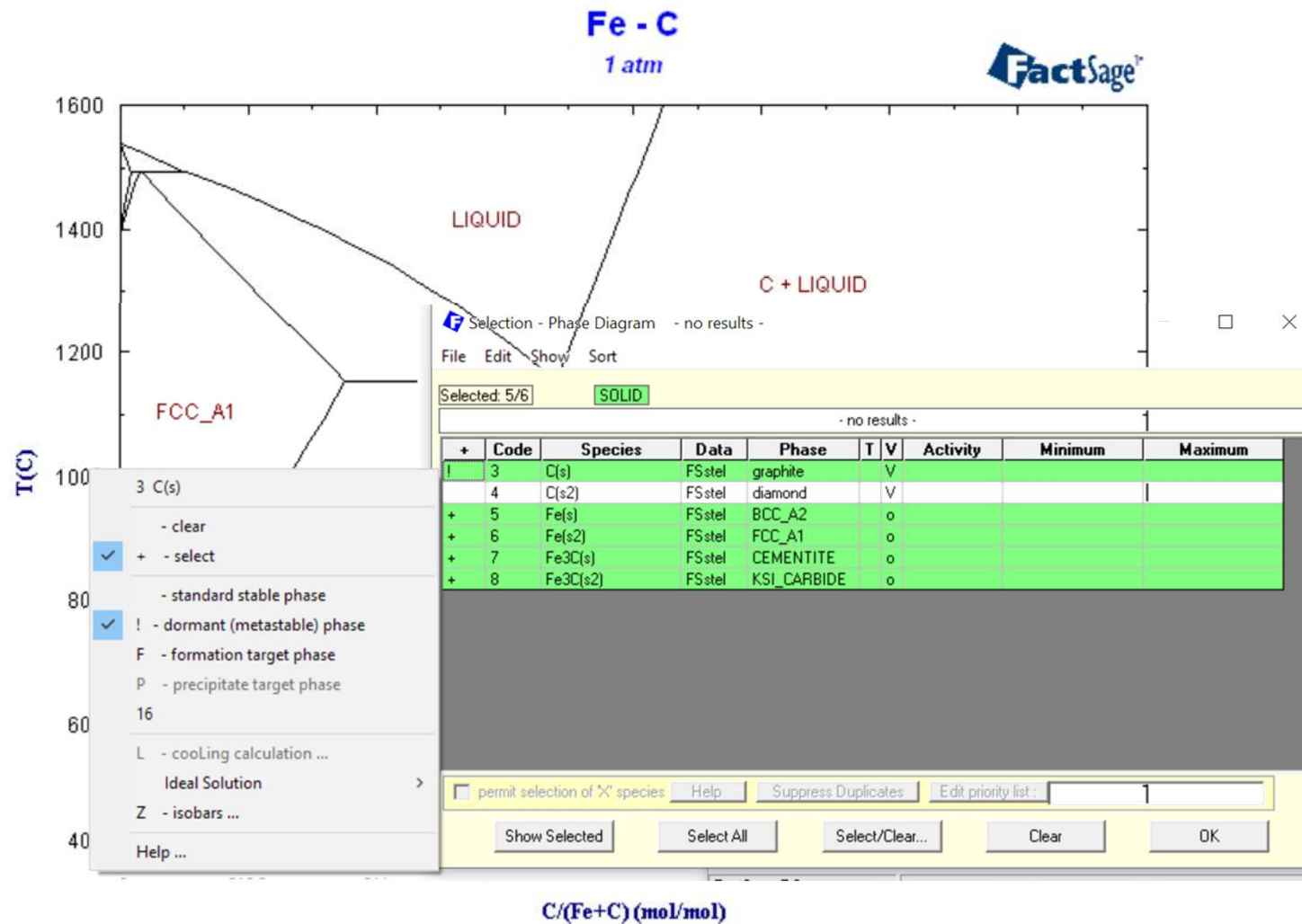
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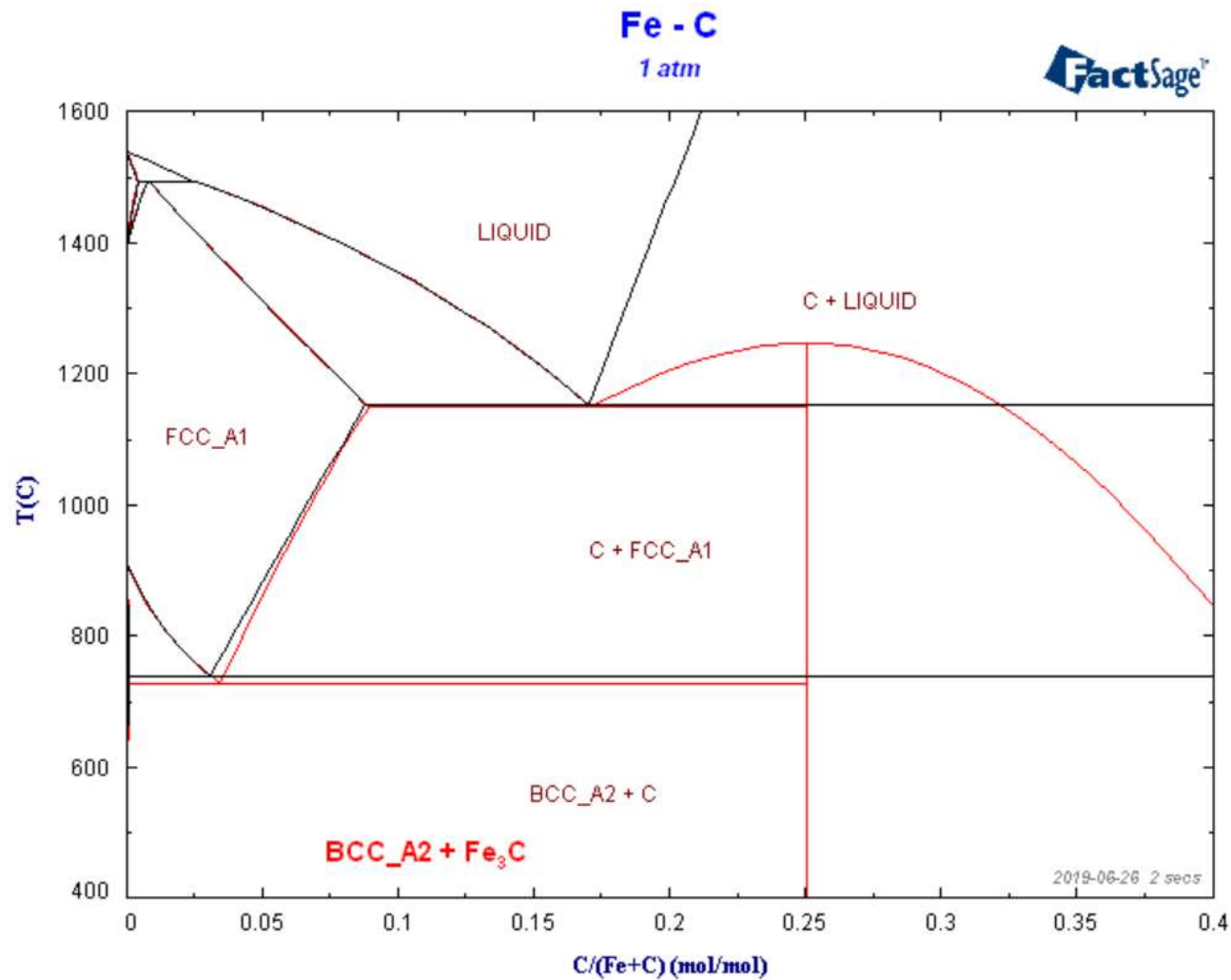
Dormant species



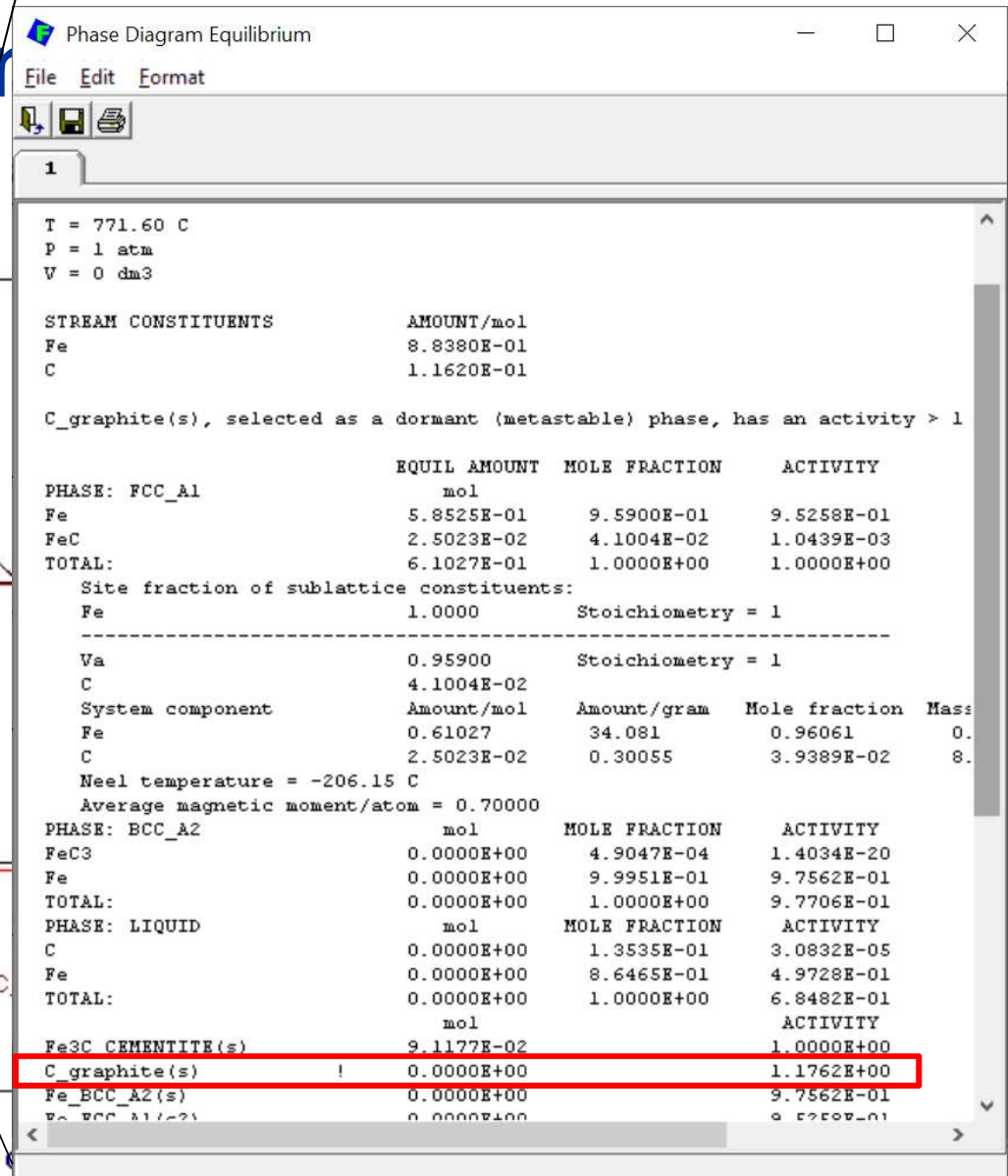
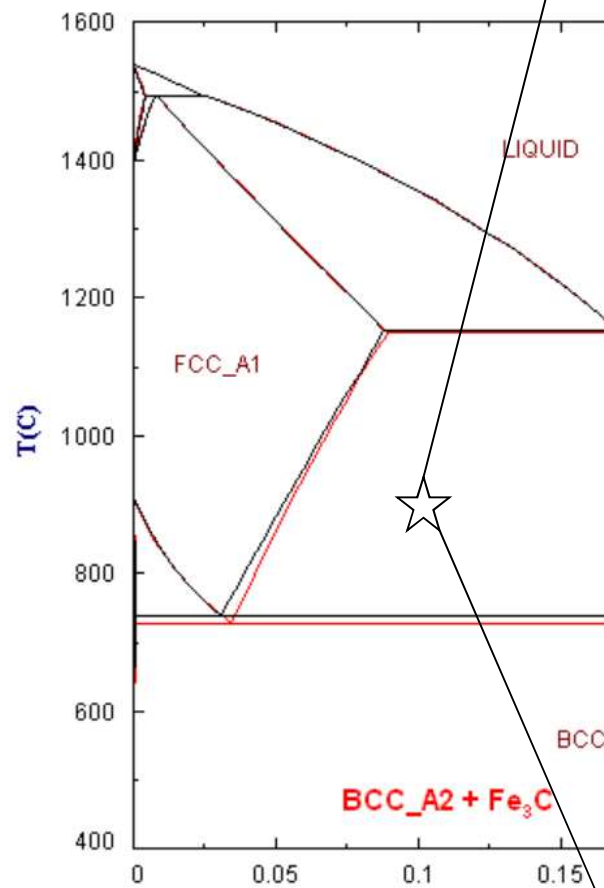
Dormant species



Dormant species



Dormar



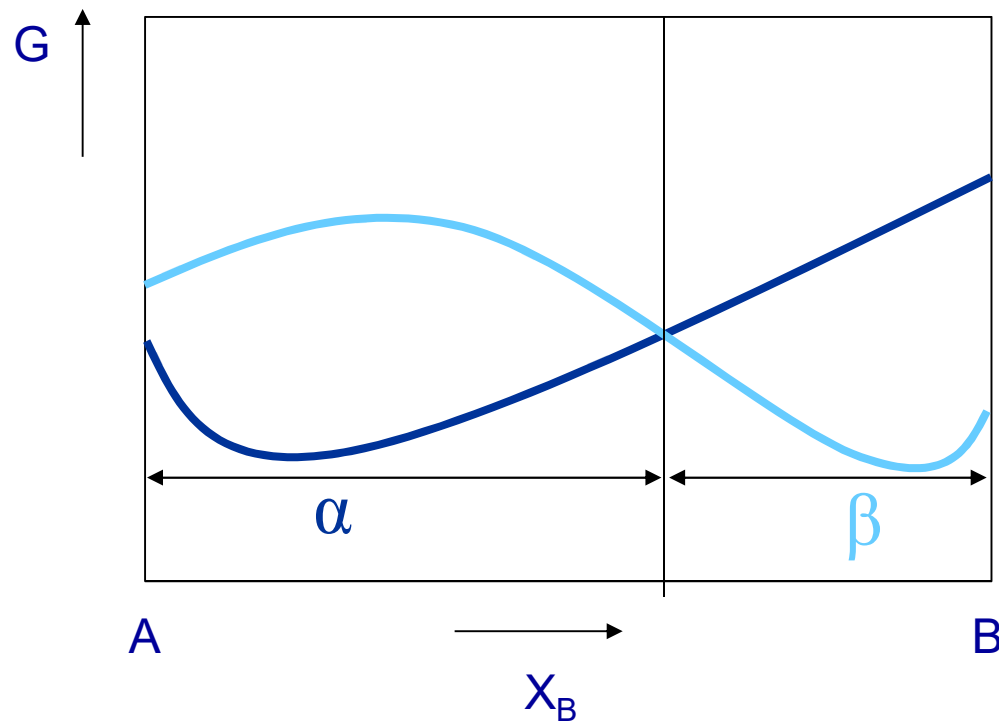
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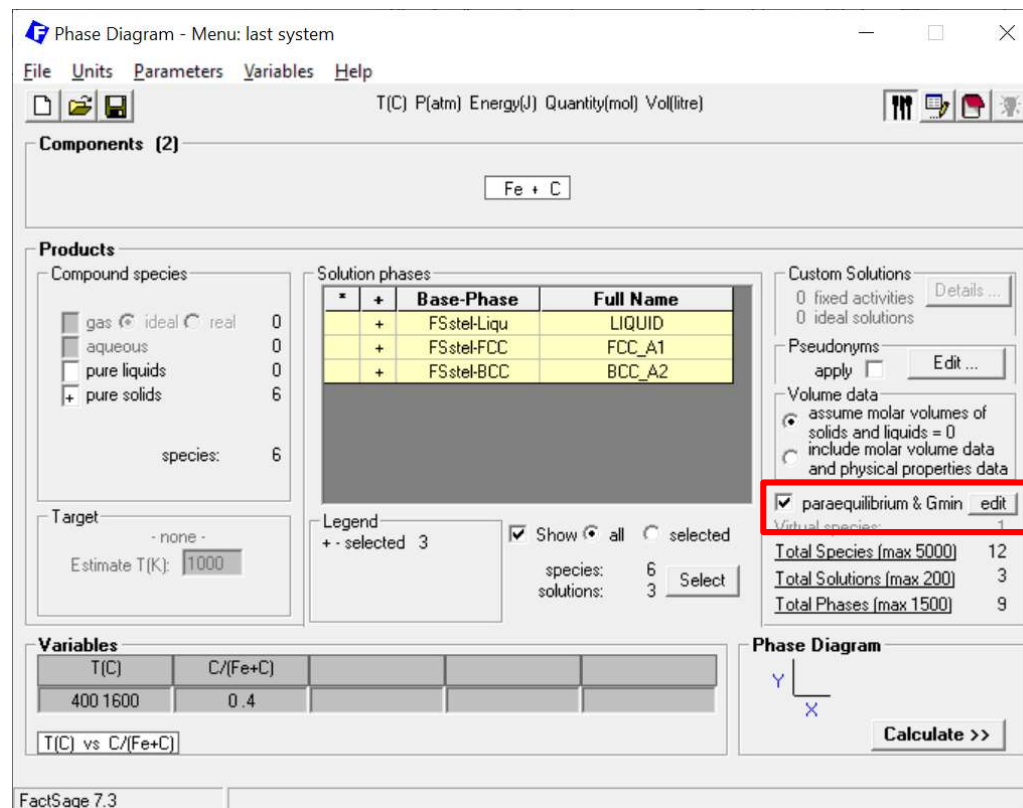
Paraequilibrium (no diffusing elements)

- In paraequilibrium, the single phase with lowest Gibbs energy forms

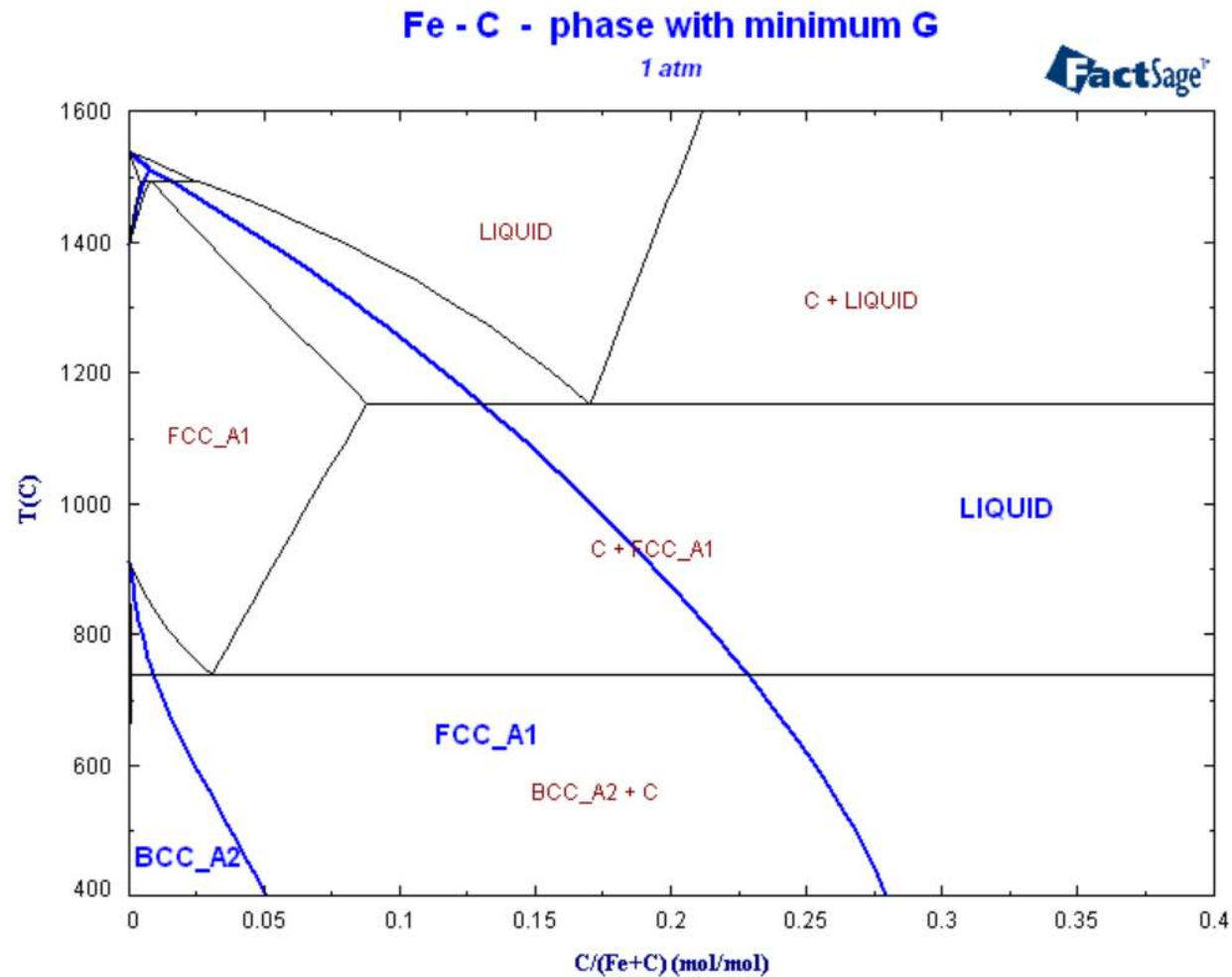


Paraequilibrium (no diffusing elements)

- In paraequilibrium, the single phase with lowest Gibbs energy forms

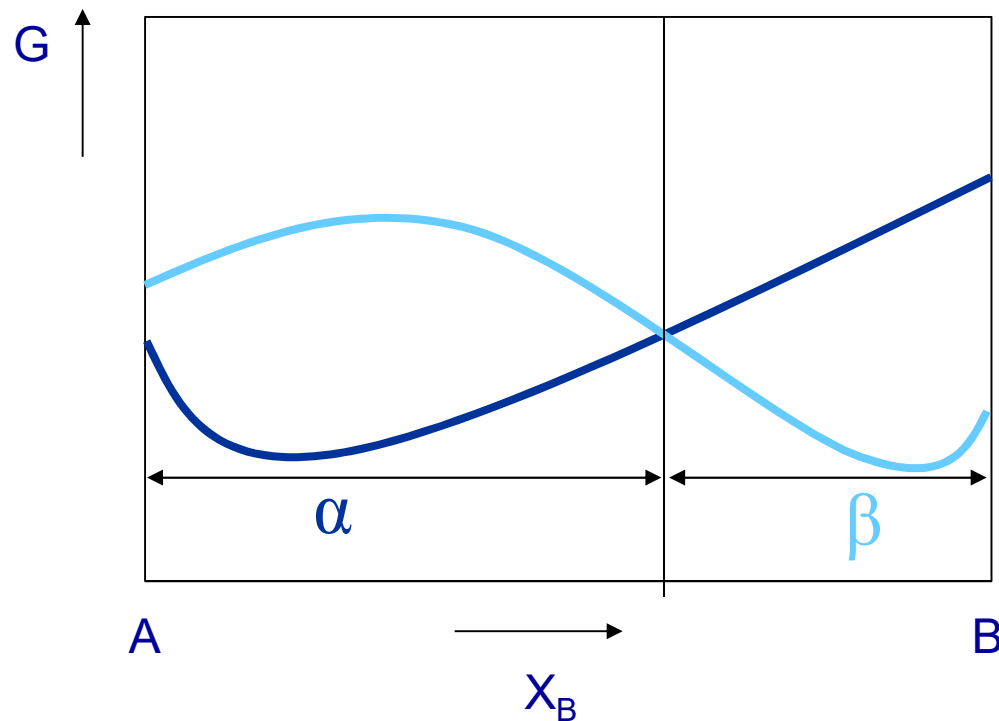


Paraequilibrium (no diffusing elements)

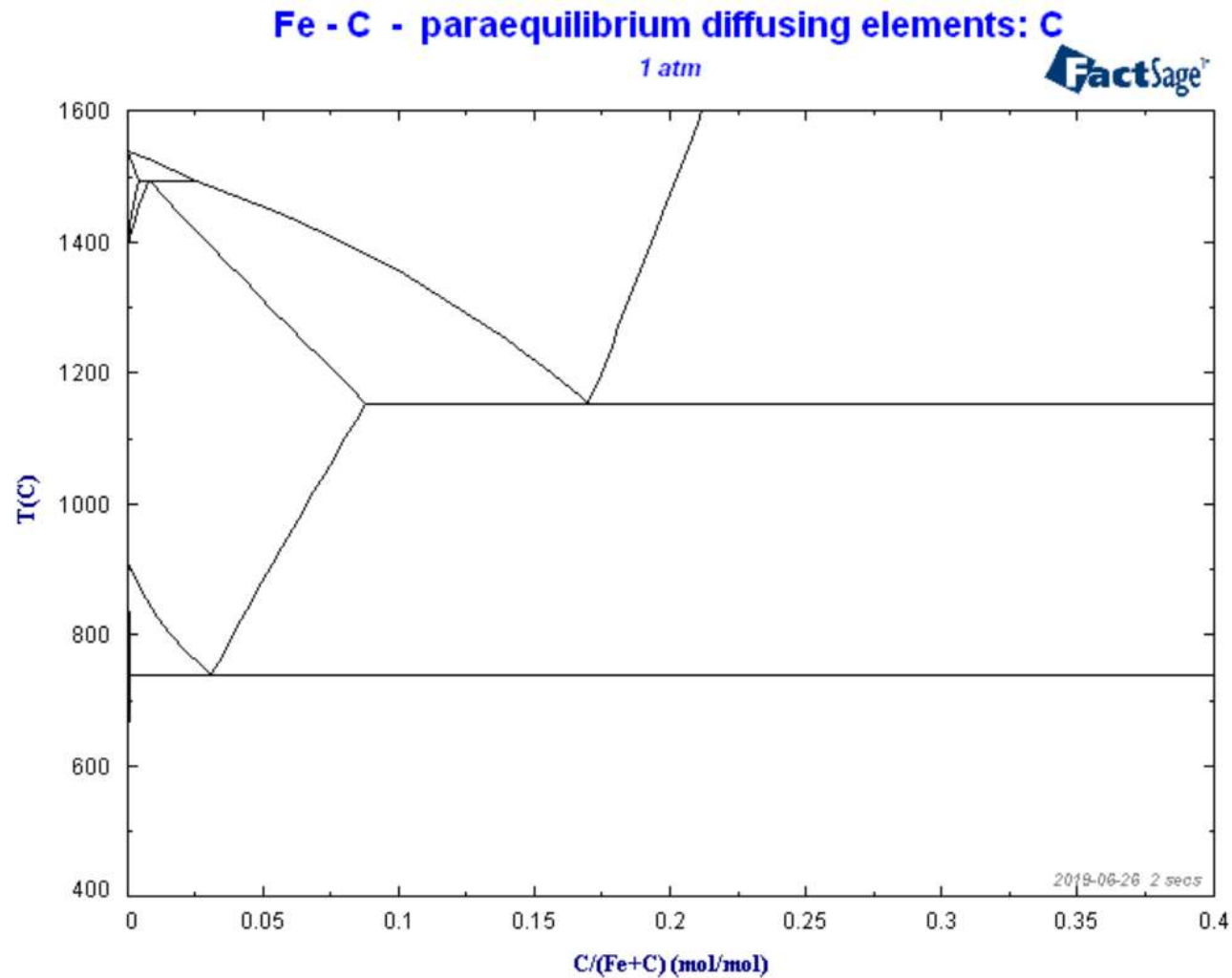


Paraequilibrium (diffusing elements)

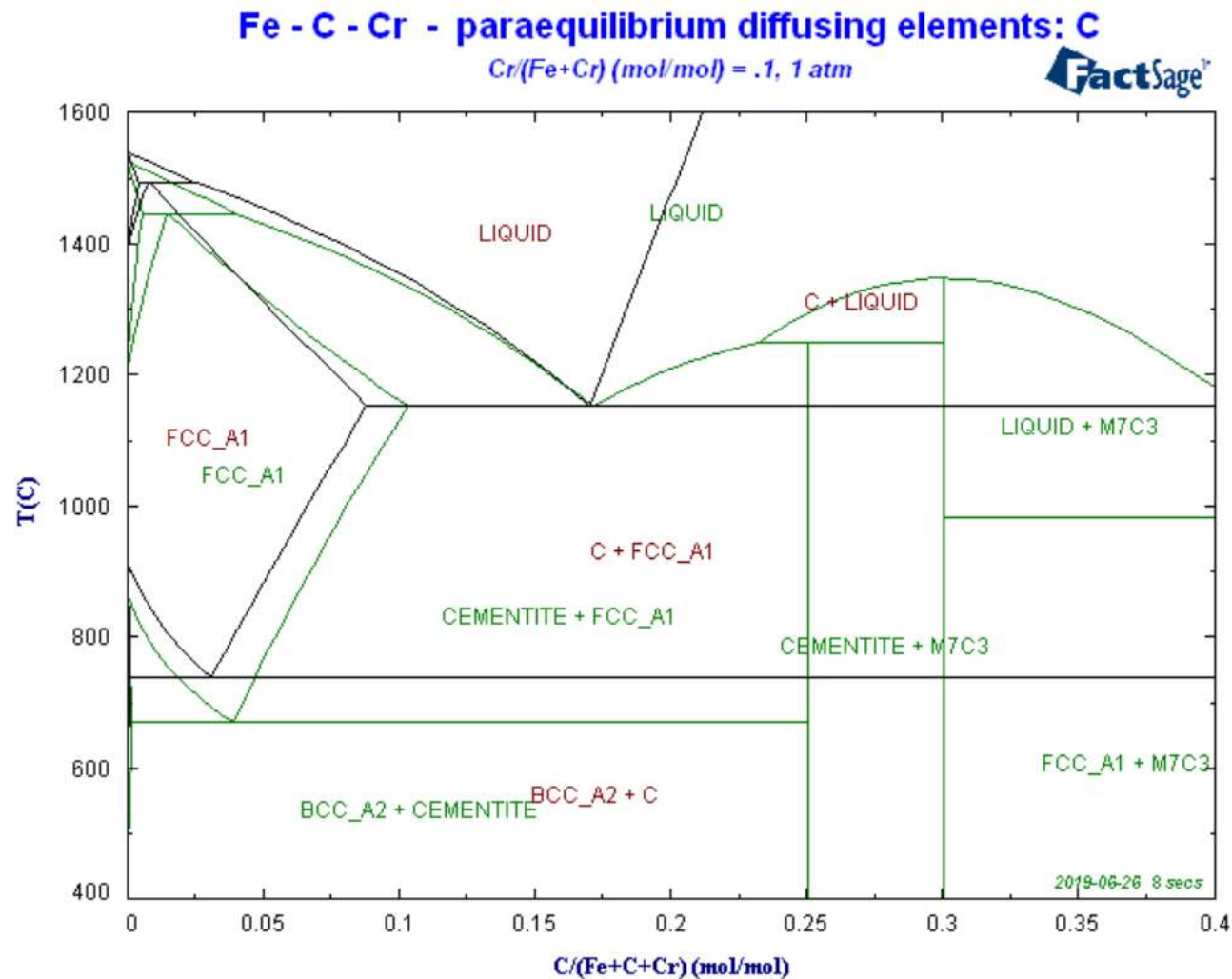
- One element (or more) allowed to partition
- Usually used for C or N



Paraequilibrium (diffusing elements)



Paraequilibrium (diffusing elements)



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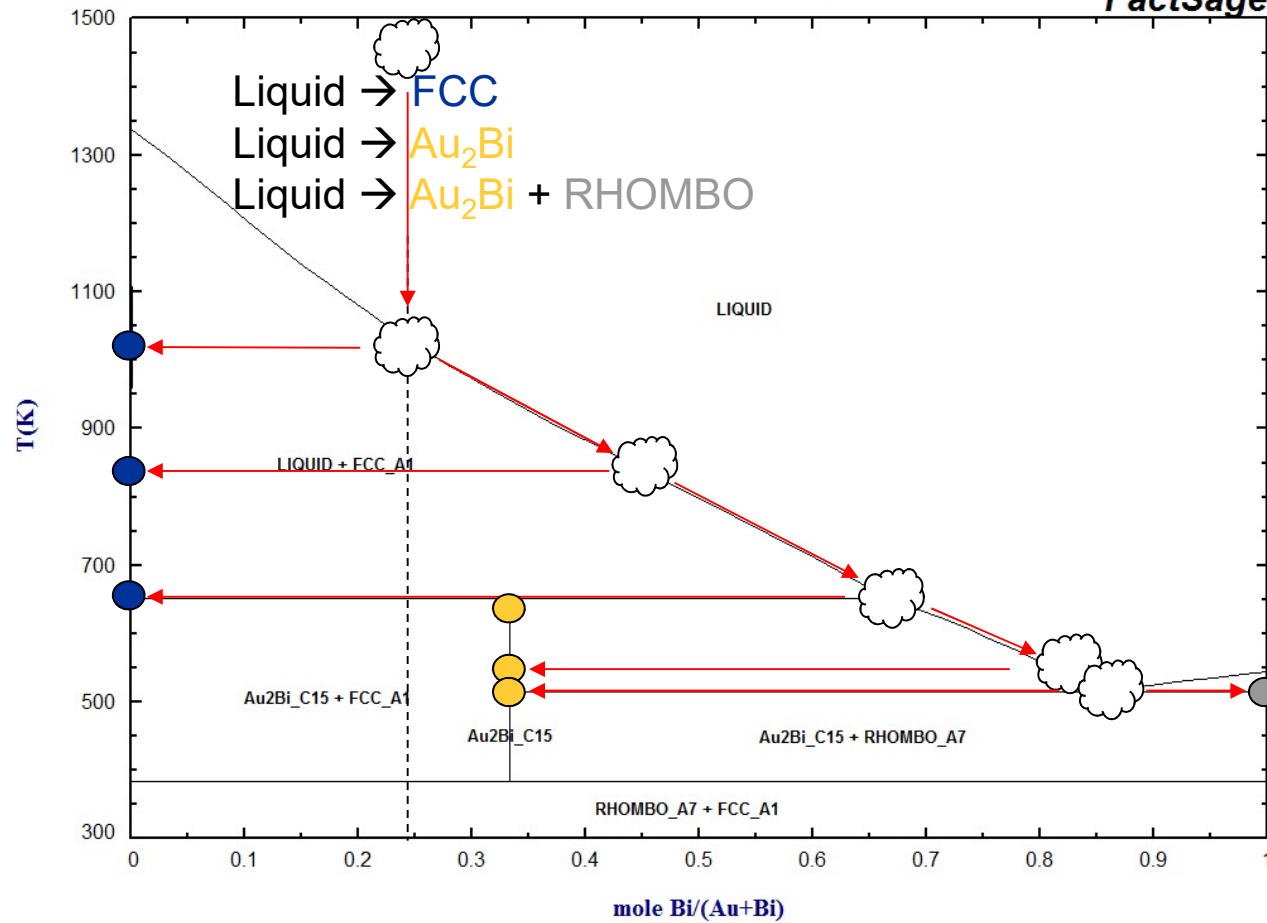


2 component Scheil cooling

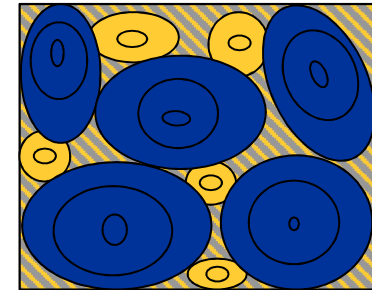
Au - Bi

Data from SGTE solders database (revised 2008)

FactSage



microstructure:

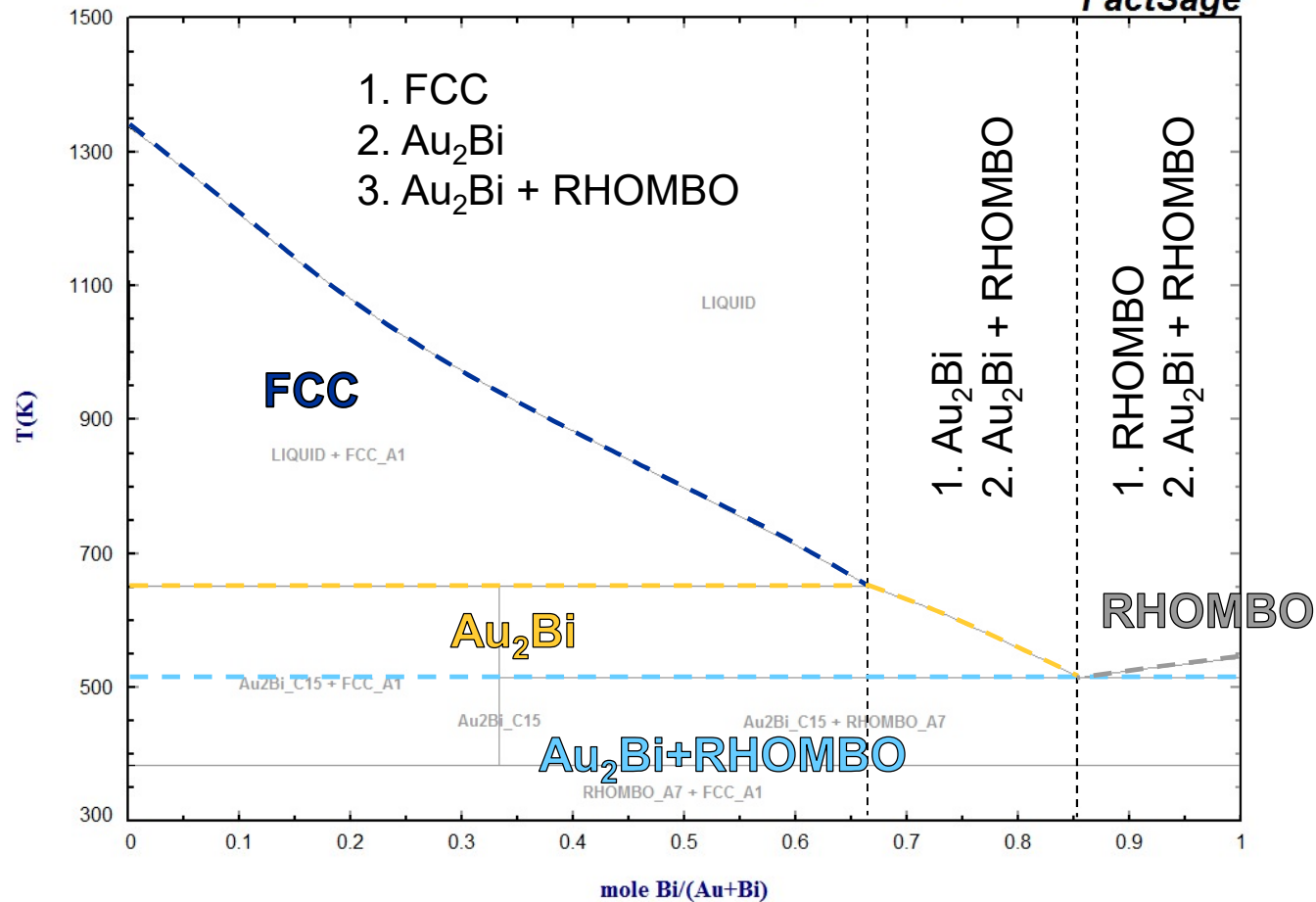


2 component Scheil cooling

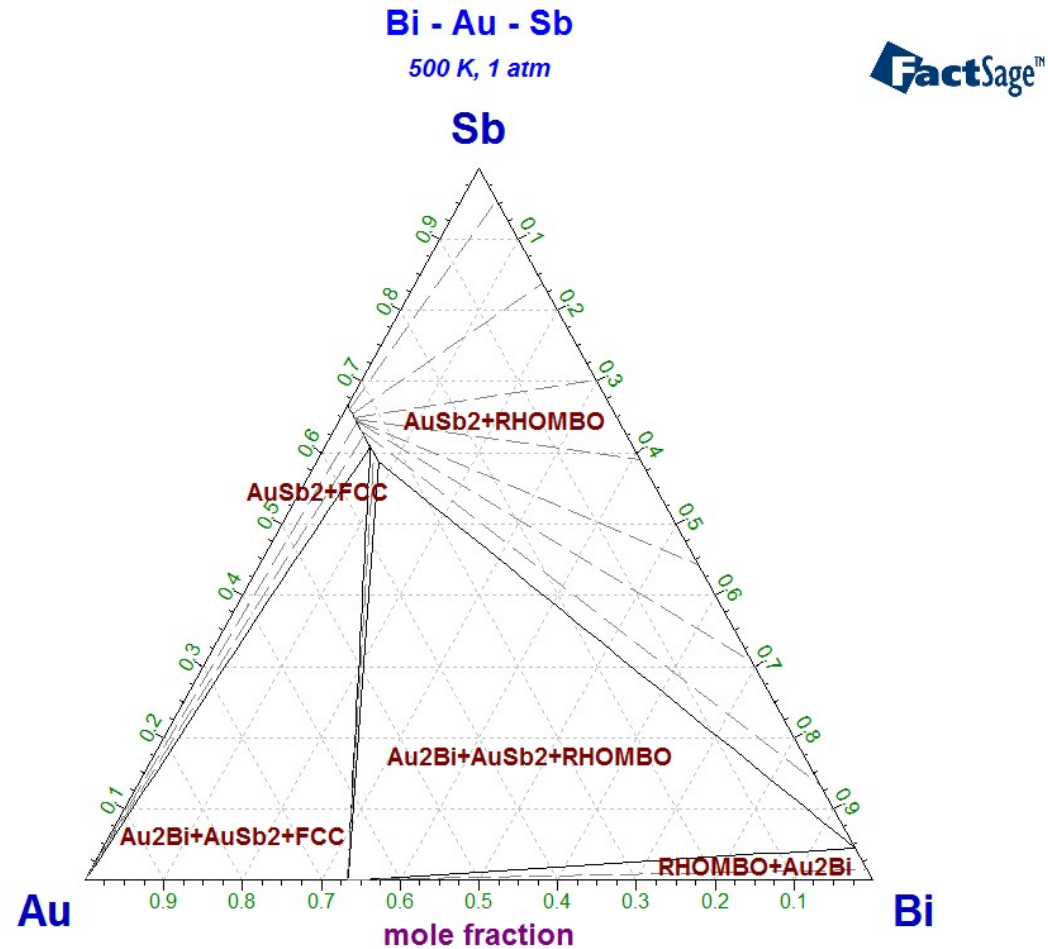
Au - Bi

Data from SGTE solders database (revised 2008)

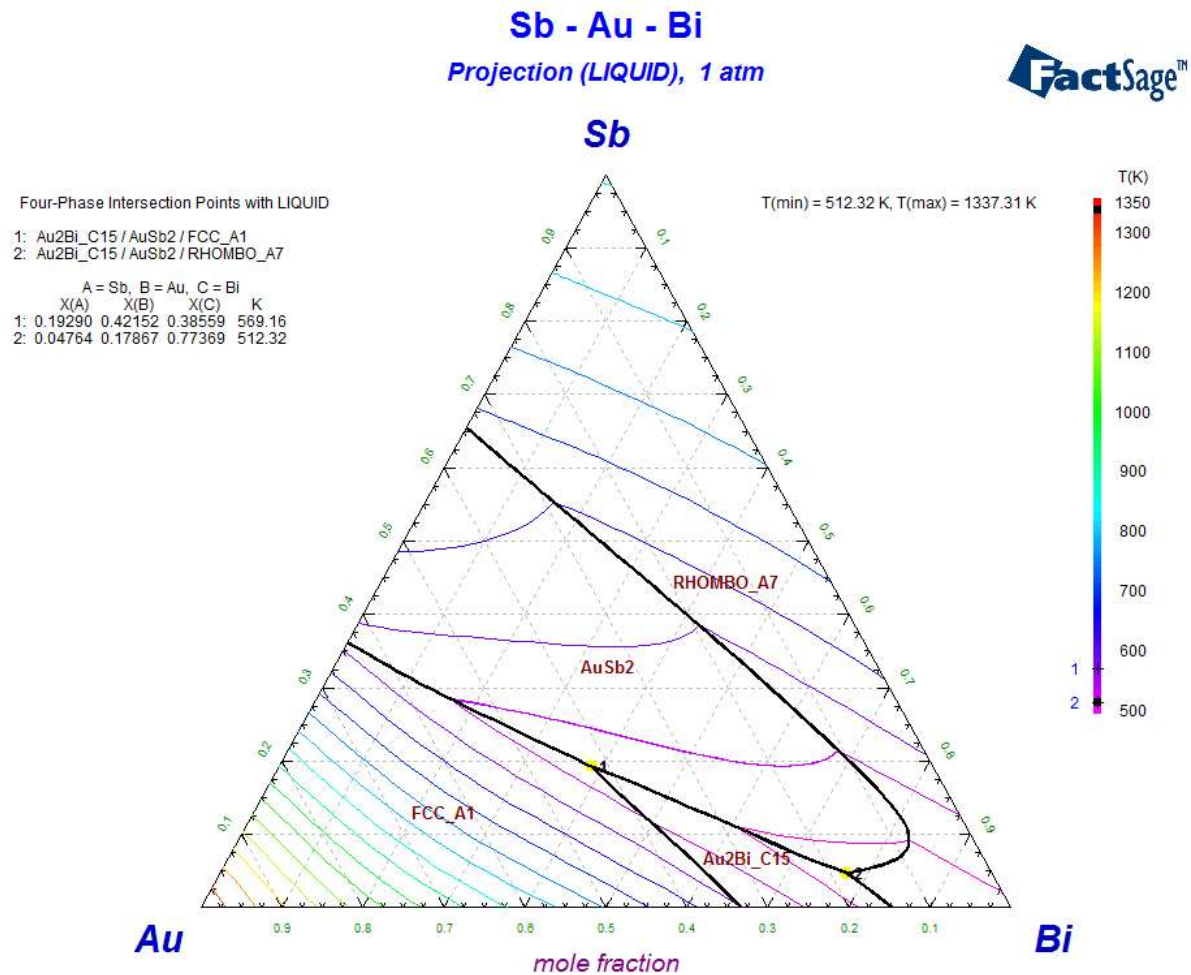
FactSage



3 components: What are the stable solid phases?



3 components: What is the first precipitating phase? → liquidus!



3 components: Scheil cooling

Sb70-Au26-Bi4

SUMMARY OF REACTIONS

770.86 to 723.46 K:

LIQ → RHOMBO

723.46 to 538.48 K:

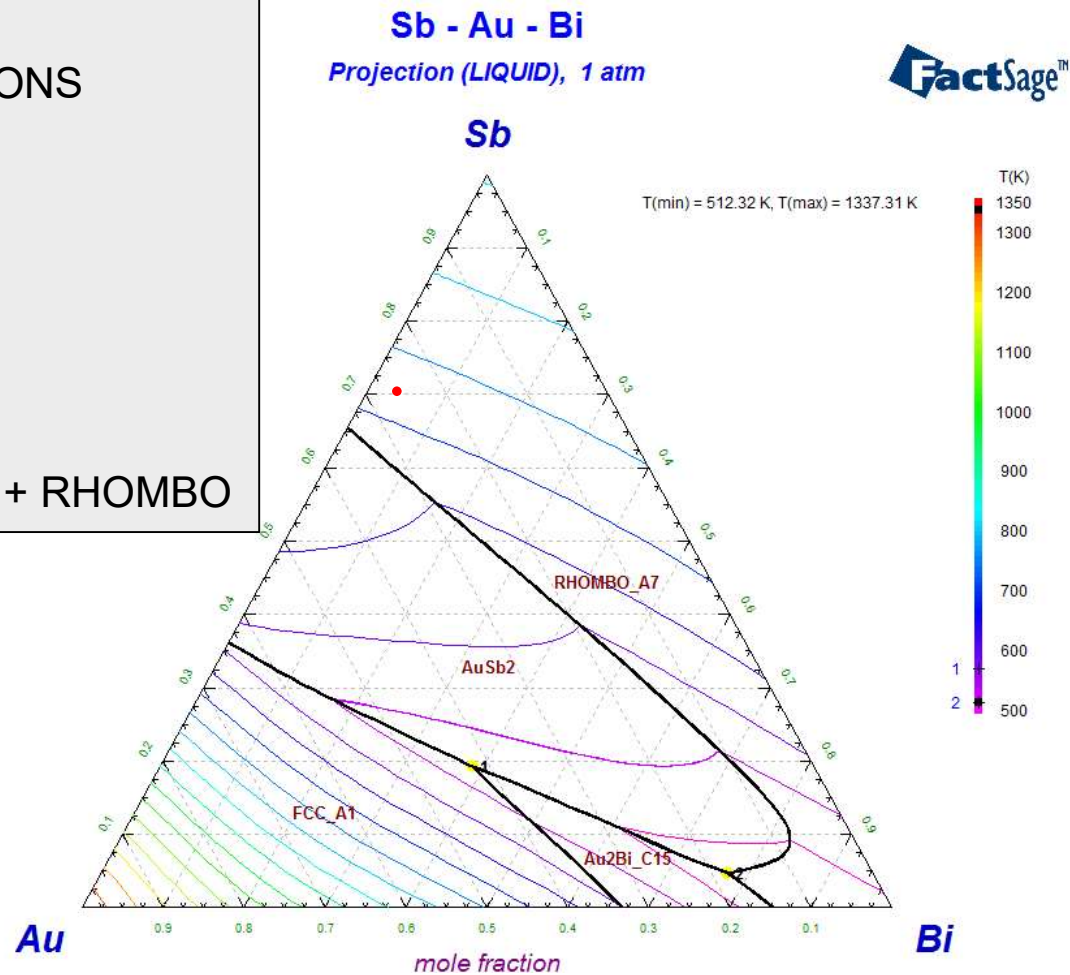
LIQ → AuSb₂

538.48 to 512.32 K:

LIQ → Au₂Bi + AuSb₂

512.32 K (isothermal):

LIQ → Au₂Bi + AuSb₂ + RHOMBO



3 components: Scheil cooling

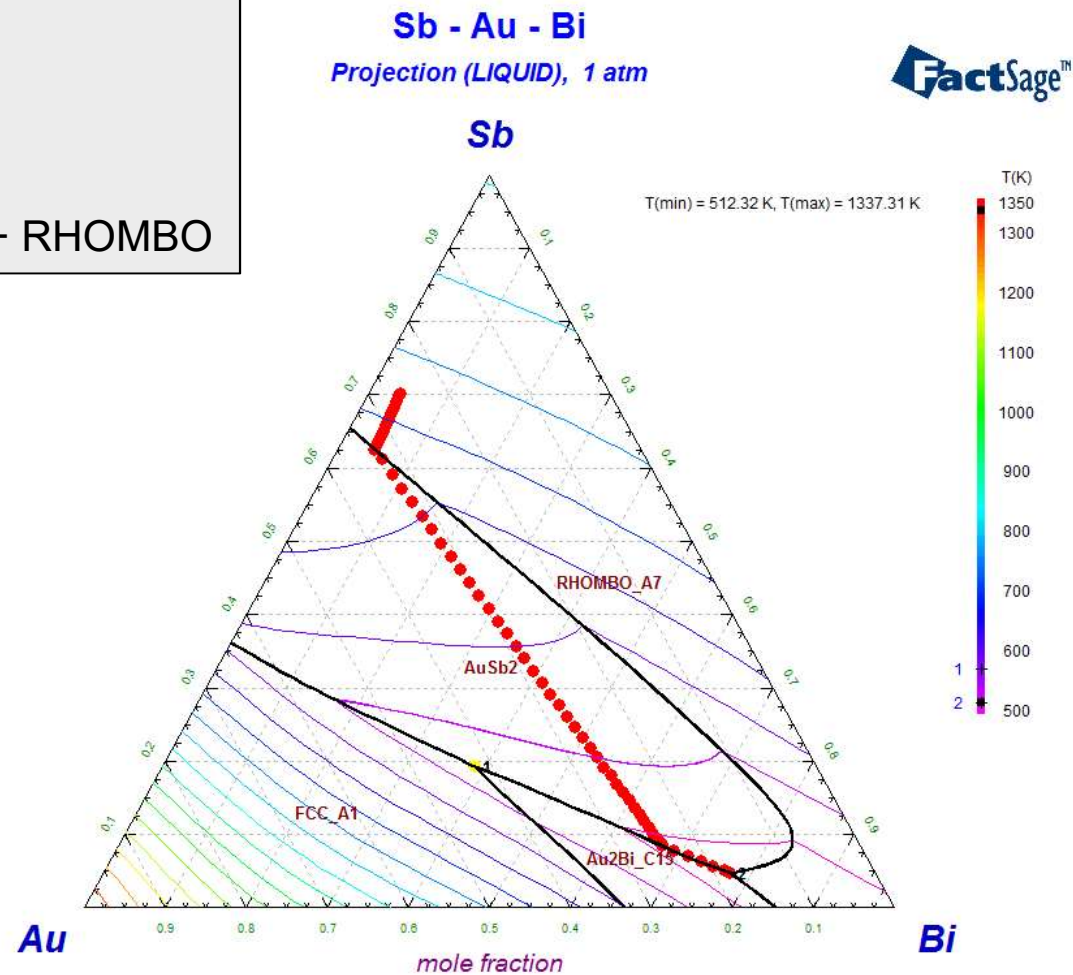
Sb70-Au26-Bi4

LIQ $\rightarrow$ RHOMBO

LIQ $\rightarrow$ AuSb₂

LIQ $\rightarrow$ Au₂Bi + AuSb₂

LIQ $\rightarrow$ Au₂Bi + AuSb₂ + RHOMBO



3 components: Scheil cooling

Sb70-Au26-Bi4

LIQ $\rightarrow$ RHOMBO

LIQ $\rightarrow$ AuSb₂

LIQ $\rightarrow$ Au₂Bi + AuSb₂

LIQ $\rightarrow$ Au₂Bi + AuSb₂ + RHOMBO

Sb70-Au28.5-Bi1.5

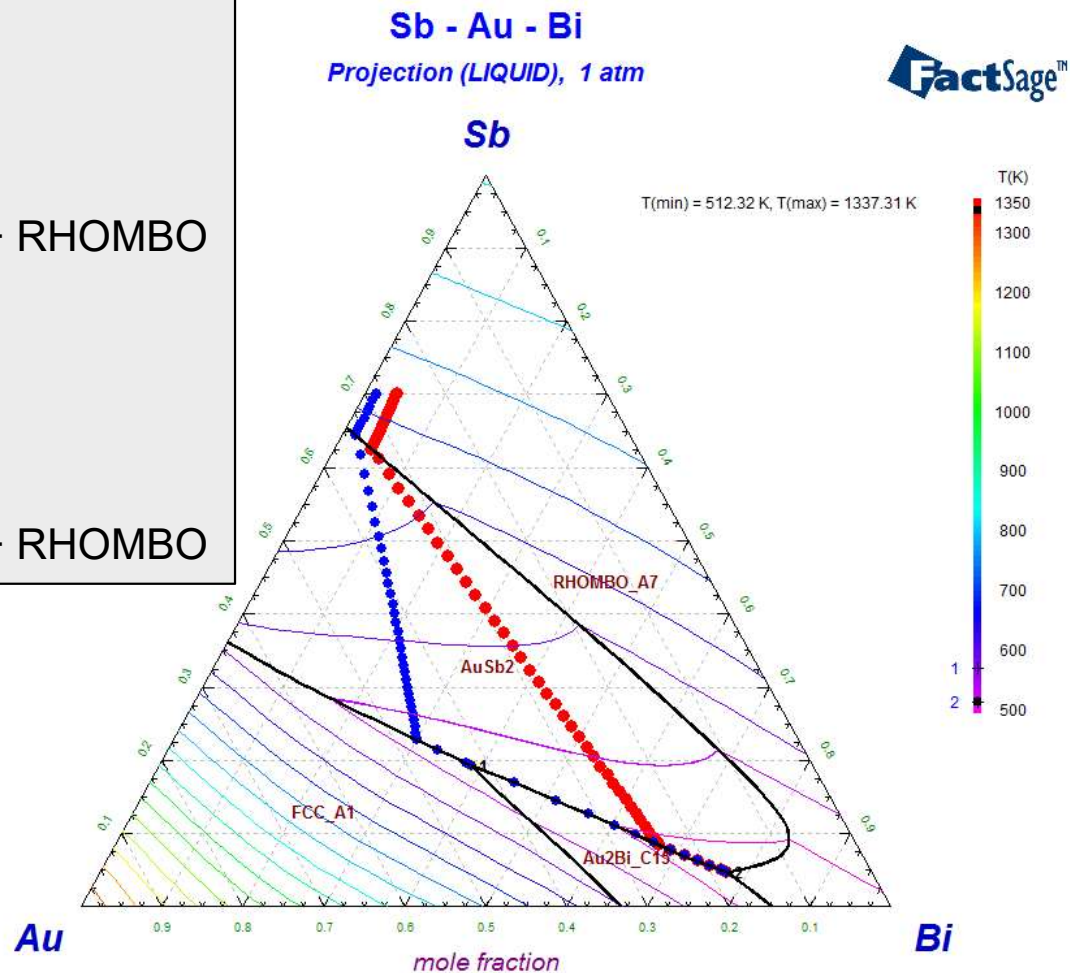
LIQ $\rightarrow$ RHOMBO

LIQ $\rightarrow$ AuSb₂

LIQ $\rightarrow$ FCC + AuSb₂

LIQ $\rightarrow$ Au₂Bi + AuSb₂

LIQ $\rightarrow$ Au₂Bi + AuSb₂ + RHOMBO



3 components: Scheil cooling

Sb70-Au26-Bi4

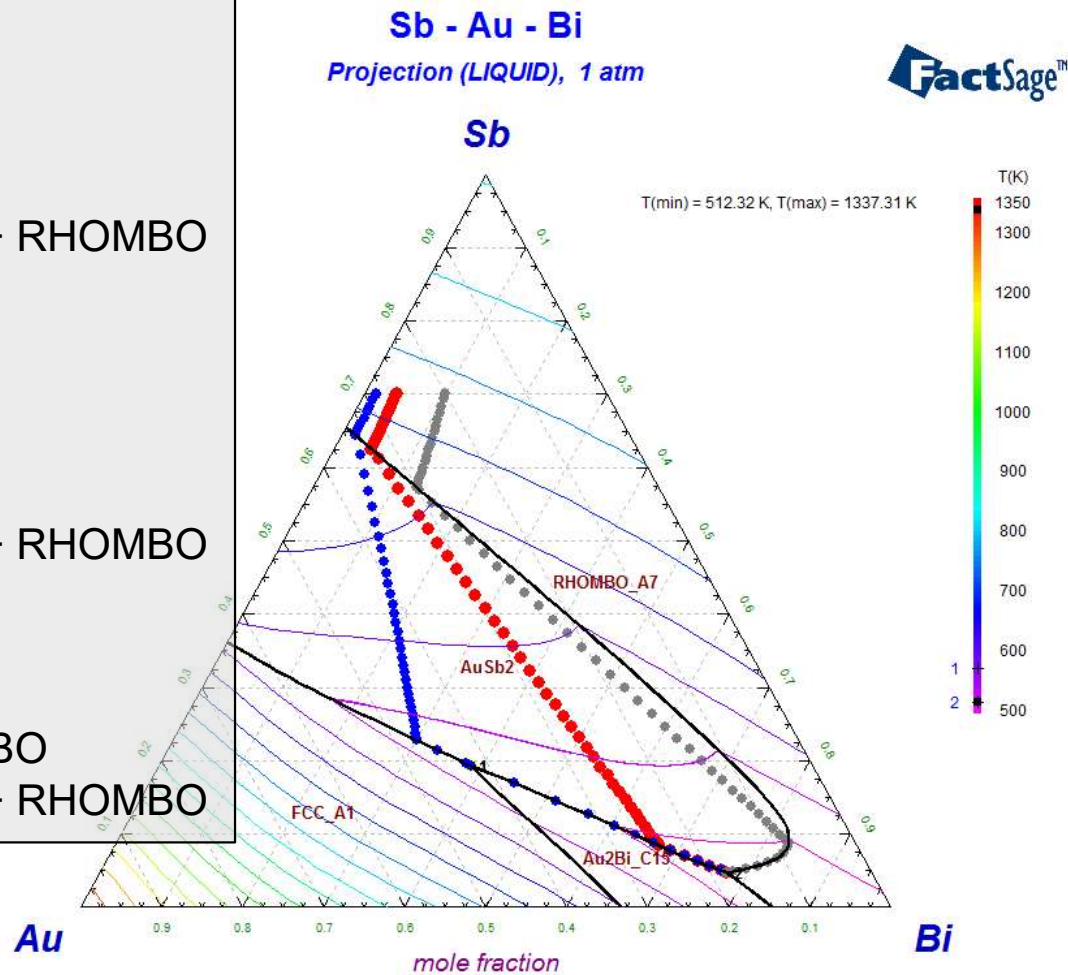
LIQ → RHOMBO
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 LIQ → AuSb₂ + Au₂Bi
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Sb70-Au28.5-Bi1.5

LIQ → RHOMBO
 LIQ → AuSb₂
 LIQ → AuSb₂ + FCC
 LIQ → AuSb₂ + Au₂Bi
 LIQ → AuSb₂ + Au₂Bi + RHOMBO

Sb70-Au20-Bi10

LIQ → RHOMBO
 LIQ → AuSb₂
 LIQ → AuSb₂ + RHOMBO
 LIQ → AuSb₂ + Au₂Bi + RHOMBO



3 components: Scheil cooling

Sb70-Au26-Bi4

LIQ → RHOMBO
 LIQ → AuSb₂
 LIQ → AuSb₂ + Au₂Bi
 LIQ → AuSb₂ + Au₂Bi + RHOMBO

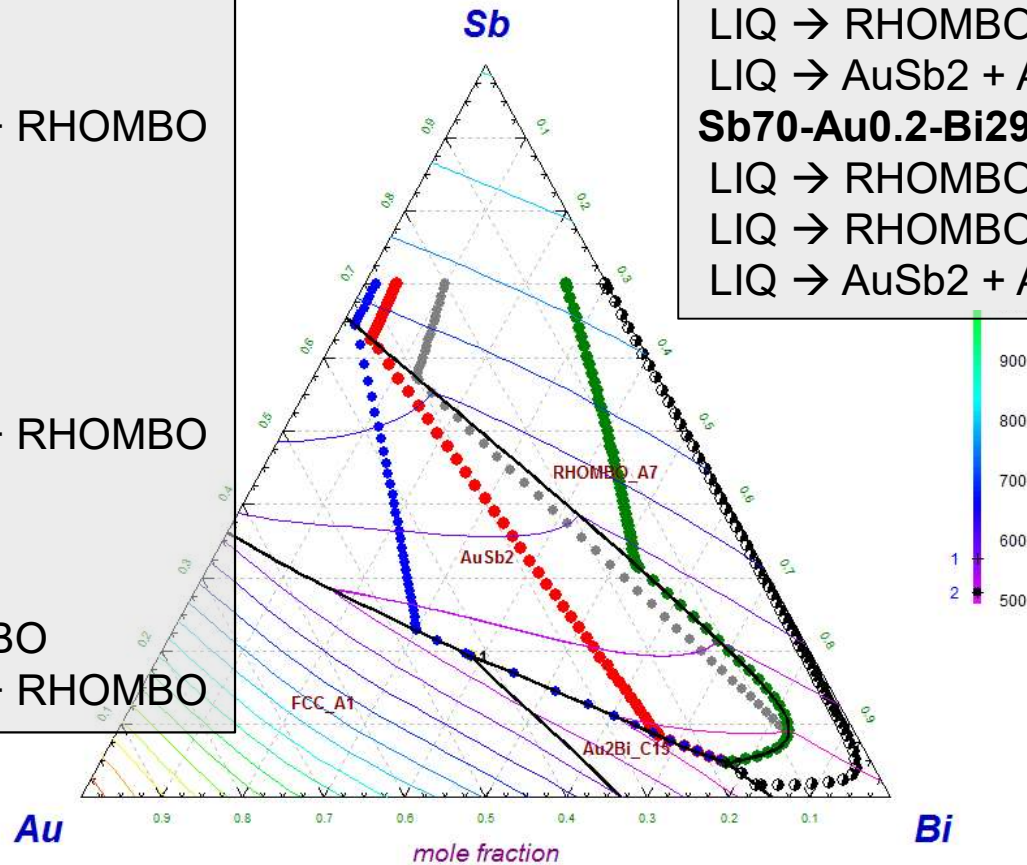
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 LIQ → AuSb₂ + RHOMBO
 LIQ → AuSb₂ + Au₂Bi + RHOMBO

Sb - Au - Bi
 Projection (LIQUID), 1 atm



Sb70-Au5-Bi25

LIQ → RHOMBO
 LIQ → RHOMBO + AuSb₂
 LIQ → AuSb₂ + Au₂Bi + RHOMBO

Sb70-Au0.2-Bi29.8

LIQ → RHOMBO
 LIQ → RHOMBO + Au₂Bi
 LIQ → AuSb₂ + Au₂Bi + RHOMBO



Since FactSage7.1: 3-component Scheil mapping

Sb70-Au26-Bi4

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LIQ → AuSb₂
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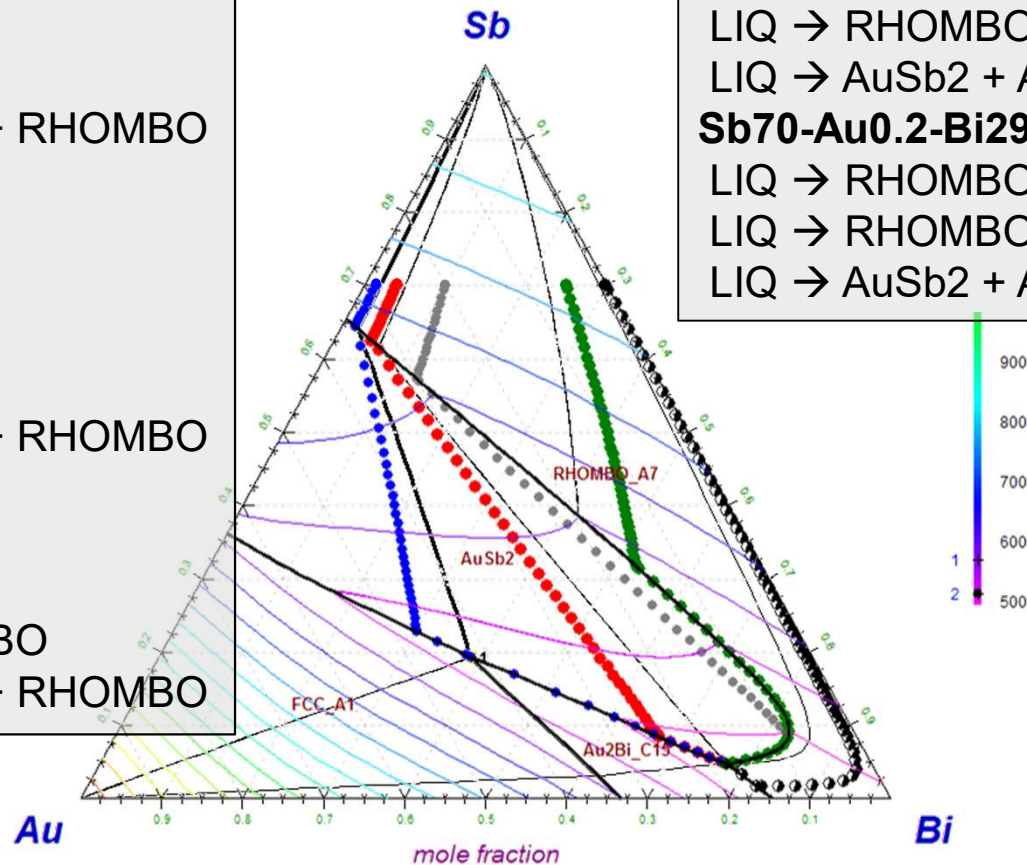
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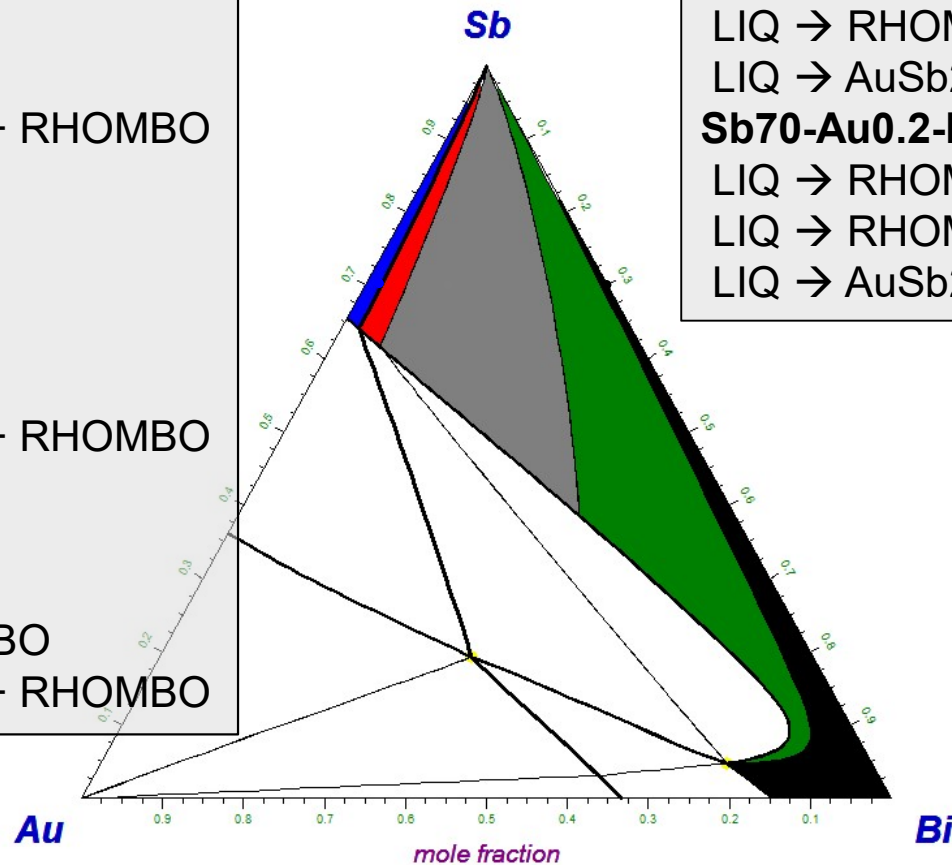
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Pertti Koukkari, *Introduction to constrained Gibbs energy methods in process and materials research*

www.vtt.fi/inf/pdf/technology/2014/T160.pdf

Risto Pajarre, *Modelling of chemical processes and materials by free energy minimization*

www.vtt.fi/inf/pdf/science/2016/S141.pdf



Immaterial system components

- In a full equilibrium calculation, acetic acid would react to $\text{CO}_2 + \text{CH}_4$:
- However, acetic acid does not decompose spontaneously at room temperature.

```

T = 25 C
P = 1 bar
V = 50.779 dm3

```

STREAM CONSTITUENTS	AMOUNT/mol
CH3COOH	1.0000E+00
H2O	5.5508E+01

PHASE: gas ideal	EQUIL AMOUNT mol	MOLE FRACTION	FUGACITY bar
CH4	9.9933E-01	4.8786E-01	4.8786E-01
CO2	9.8402E-01	4.8038E-01	4.8038E-01
H2O	6.5053E-02	3.1758E-02	3.1758E-02
H2	3.9065E-06	1.9071E-06	1.9071E-06
C2H6	2.3499E-07	1.1472E-07	1.1472E-07
CO	5.6128E-10	2.7401E-10	2.7401E-10
TOTAL:	2.0484E+00	1.0000E+00	1.0000E+00

System component	Amount/mol	Amount/gram	Mole fraction
O	2.0331	32.528	0.24965
C	1.9834	23.821	0.24354
H	4.1274	4.1602	0.50681

PHASE: aqueous	mol	MOLALITY	ACTIVITY
H2O_liquid	5.5443E+01	5.5508E+01	9.9970E-01
H[+]	8.1229E-05	8.1325E-05	8.1325E-05
H2	1.5955E-09	1.5974E-09	1.5974E-09
CH4	6.6978E-04	6.7057E-04	6.7057E-04
C2H2	5.2788E-40	5.2850E-40	5.2850E-40
C2H4	6.1309E-22	6.1381E-22	6.1381E-22
C2H6	1.9969E-10	1.9992E-10	1.9992E-10
CH3COO[-]	1.5757E-10	1.5776E-10	1.5776E-10
CH3COOH	6.9564E-10	6.9646E-10	6.9646E-10



Immaterial system components

- In a full equilibrium calculation, acetic acid would react to $\text{CO}_2 + \text{CH}_4$:
- However, acetic acid does not decompose spontaneously at room temperature.
- One option is to set the gas phase dormant:

```

T = 25 C
P = 1 atm
V = 0 dm3

```

STREAM CONSTITUENTS	AMOUNT/mol
CH3COOH	1.0000E+00
H2O	5.5508E+01

	EQUIL AMOUNT mol	MOLALITY	ACTIVITY
PHASE: aqueous			
H2O_liquid	5.5508E+01	5.5508E+01	9.6460E-01
H[+]	6.3390E-04	6.3391E-04	6.3391E-04
H2	3.4637E-09	3.4637E-09	3.4637E-09
CH4	9.9957E-01	9.9958E-01	9.9958E-01
C2H2	1.1519E-34	1.1519E-34	1.1519E-34
C2H4	2.9008E-16	2.9009E-16	2.9009E-16
C2H6	2.0487E-04	2.0487E-04	2.0487E-04
CH3COO[-]	1.8940E-06	1.8940E-06	1.8940E-06
CH3COOH	6.5176E-05	6.5177E-05	6.5177E-05
HC03[-]	6.3200E-04	6.3201E-04	6.3201E-04
HC204[-]	7.8563E-16	7.8564E-16	7.8564E-16
TOTAL:	5.7508E+01		1.0000E+00



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```

STREAM CONSTITUENTS	AMOUNT/mol		
CH3COOH	1.0000E+00		
H2O	5.5508E+01		

	EQUIL AMOUNT mol	MOLALITY	ACTIVITY
PHASE: aqueous			
H2O_liquid	5.5508E+01	5.5508E+01	9.6460E-01
H[+]	6.3390E-04	6.3391E-04	6.3391E-04
H2	3.4637E-09	3.4637E-09	3.4637E-09
CH4	9.9957E-01	9.9958E-01	9.9958E-01
C2H2	1.1519E-34	1.1519E-34	1.1519E-34
C2H4	2.9008E-16	2.9009E-16	2.9009E-16
C2H6	2.0487E-04	2.0487E-04	2.0487E-04
CH3COO[-]	1.8940E-06	1.8940E-06	1.8940E-06
CH3COOH	6.5176E-05	6.5177E-05	6.5177E-05
HC03[-]	6.3200E-04	6.3201E-04	6.3201E-04
HC204[-]	7.8563E-16	7.8564E-16	7.8564E-16
TOTAL:	5.7508E+01		1.0000E+00

We need to introduce a real constraint CH_3COOH may ONLY dissociate!



Immaterial system components

- Therefore, the stoichiometric matrix of the aqueous solution species is adjusted:

Species	H	O	C	e(aq)	V
H ₂ O	2	1	0	0	0
H[+]	1	0	0	-1	0
OH[-]	1	1	0	1	0
...					
CH ₃ COO[-]	3	2	2	1	1
CH ₃ COOH	4	2	2	0	1



Immaterial system components

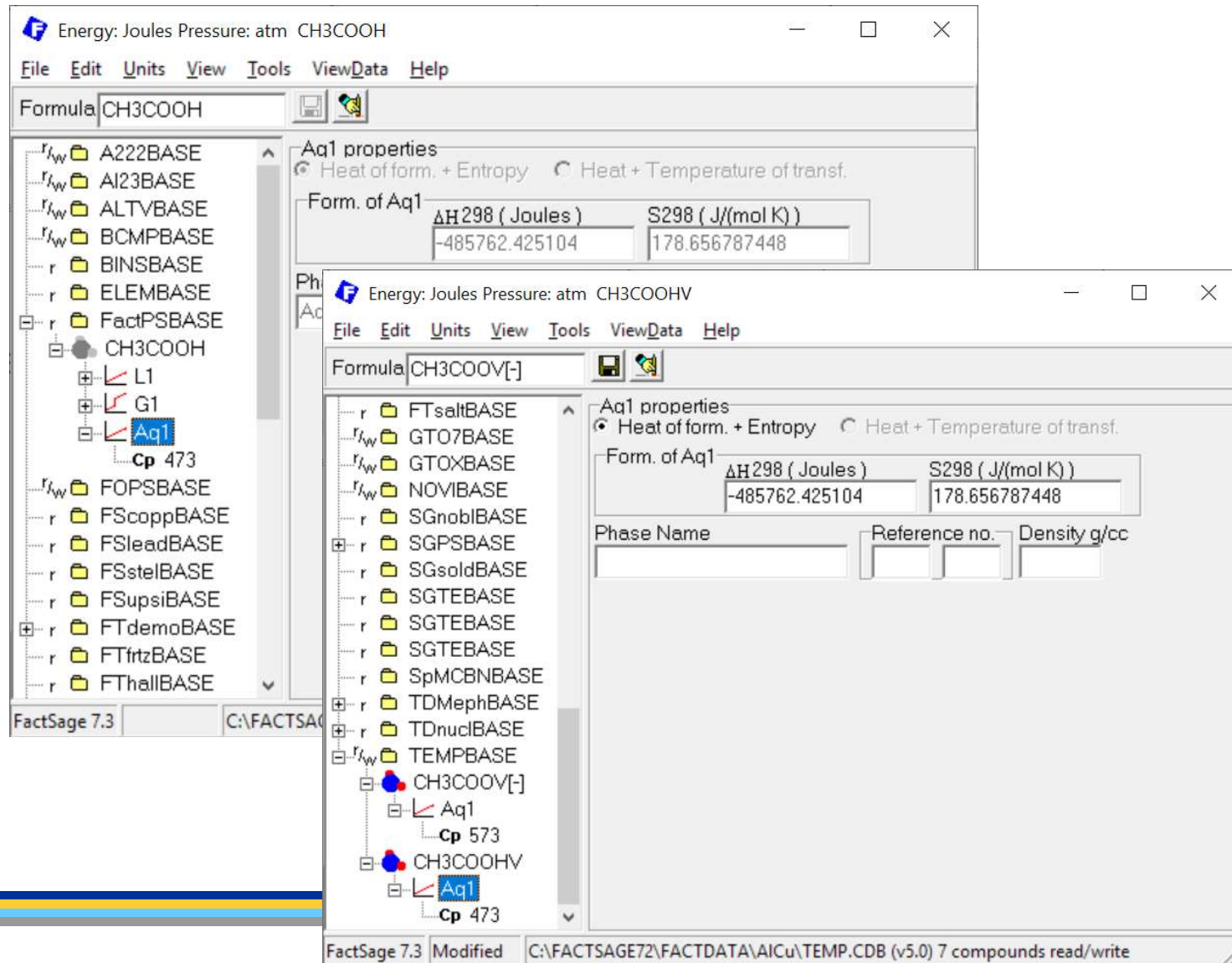
The screenshot displays the FactSage 7.3 software window. The title bar indicates the current state: "Energy: Joules Pressure: atm CH3COOH". The menu bar includes File, Edit, Units, View, Tools, ViewData, and Help. The Formula field contains "CH3COOH". On the left, a tree view shows various databases, with "CH3COOH" expanded to show "Aq1" selected. The right pane, titled "Aq1 properties", shows two radio buttons: "Heat of form. + Entropy" (selected) and "Heat + Temperature of transf.". Below these, a table for "Form. of Aq1" displays thermodynamic data:

Form. of Aq1	ΔH_{298} (Joules)	S_{298} (J/(mol K))
	-485762.425104	178.656787448

Below the table, a section for "Aqueous" properties shows "Phase Name" as "Aqueous", "Reference no." as "7", and "Density g/cc" as an empty field. The status bar at the bottom indicates "FactSage 7.3" and the file path "C:\FACTSAGE73\FACTDATA\F553base.cdb (v7.30) 4920 compounds read-only".



Immaterial system components



The image displays two overlapping windows from the FactSage 7.3 software, showing thermodynamic data for different chemical species.

Left Window: CH3COOH

- Formula: CH3COOH
- Energy: Joules Pressure: atm
- File Edit Units View Tools ViewData Help
- Form. of Aq1: ΔH_{298} (Joules) = -485762.425104, S_{298} (J/(mol K)) = 178.656787448
- Phases: Aq1 properties (Heat of form. + Entropy selected)
- Tree view: Aq1 is selected under CH3COOH.

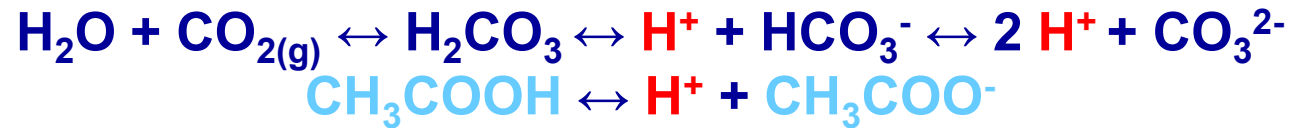
Right Window: CH3COOHV

- Formula: CH3COOHV
- Energy: Joules Pressure: atm
- File Edit Units View Tools ViewData Help
- Form. of Aq1: ΔH_{298} (Joules) = -485762.425104, S_{298} (J/(mol K)) = 178.656787448
- Phases: Aq1 properties (Heat of form. + Entropy selected)
- Tree view: Aq1 is selected under CH3COOHV.

Both windows show a list of phases on the left, including A222BASE, A123BASE, ALTVBASE, BCMPBASE, BINSBASE, ELEMBASE, FactPSBASE, CH3COOH, L1, G1, Aq1, Cp 473, FOPBASE, FScooppBASE, FSleadBASE, FSstelBASE, FSupsiBASE, FTdemoBASE, FTfritzBASE, FTallBASE, FTsaltBASE, GTO7BASE, GTOXBASE, NOVIBASE, SGnoblBASE, SGPSBASE, SGsoldBASE, SGTEBASE, SpMCBNBASE, TDMephBASE, TDnuclBASE, TEMPBASE, CH3COOHV, Aq1, Cp 573, and CH3COOH.



Example 1: Water + carbonic acid + acetic acid



Equilib - Reactants

File Edit Table Units Data Search Data Evaluation Help

T(C) P(atm) Energy(J) Quantity(mol) Vol(litre)

1 - 3

Quantity(mol)	Species	Phase	T(C)	P(total)**	Stream#	Data
1 kg	H2O				1	
+ 1	H2CO3				1	
+ <A>	CH3COOH				1	

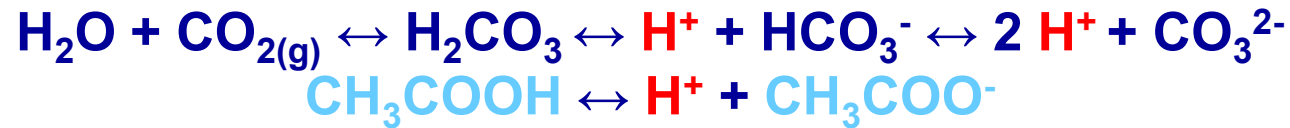
☐ Initial Conditions

Next >>

FactSage 7.3 Compound: 2/35 databases Solution: 1/34 databases



Example 1: Water + carbonic acid + acetic acid



Selection - Equilib Page 11/11 : T(C) = 25, P(atm) = 1, Alpha = 1

File Edit Show Sort

Selected: 22/30 **AQUEOUS** 11 Pages: 1 - 11 [page]

Page 11/11 : T(C) = 25, P(atm) = 1, Alpha = 1 [min = 0 at p. 1; max = 1 at p. 11]

+	Code	Species	Data	Phase	T	V	Activity	Minimum	Maximum
+	75	HC00[.](aq)	FactPS	aqueous			1.4998E-33	1.4998E-33 [11]	5.5426E-32 [1]
+	76	HC00H(aq)	FactPS	aqueous			3.3068E-32	3.3068E-32 [11]	3.3441E-32 [1]
	77	CH3C00[.](aq)	FactPS	aqueous					
	78	CH3C00H(aq)	FactPS	aqueous			8.9433E-97		
+	79	HC03[.](aq)	FactPS	aqueous			3.1263E-06	3.1263E-06 [11]	1.1623E-04 [1]
+	80	HC204[.](aq)	FactPS	aqueous			7.5232E-42	7.5232E-42 [11]	2.7786E-40 [1]
	81	VO[2+](aq)	FactPS	aqueous					
	82	VO2[+](aq)	FactPS	aqueous					
	83	VO3[.](aq)	FactPS	aqueous					
	84	VO4[.](aq)	FactPS	aqueous					
	85	HVO4[2-](aq)	FactPS	aqueous					
	86	HV10028[5-](aq)	FactPS	aqueous					
+	87	CH3C00V[.](aq)	TEMP	Aq1			4.2443E-03	0 [1]	4.2443E-03 [11]
+	88	CH3C00HV(aq)	TEMP	Aq1			0.9786	0 [1]	0.9786 [11]

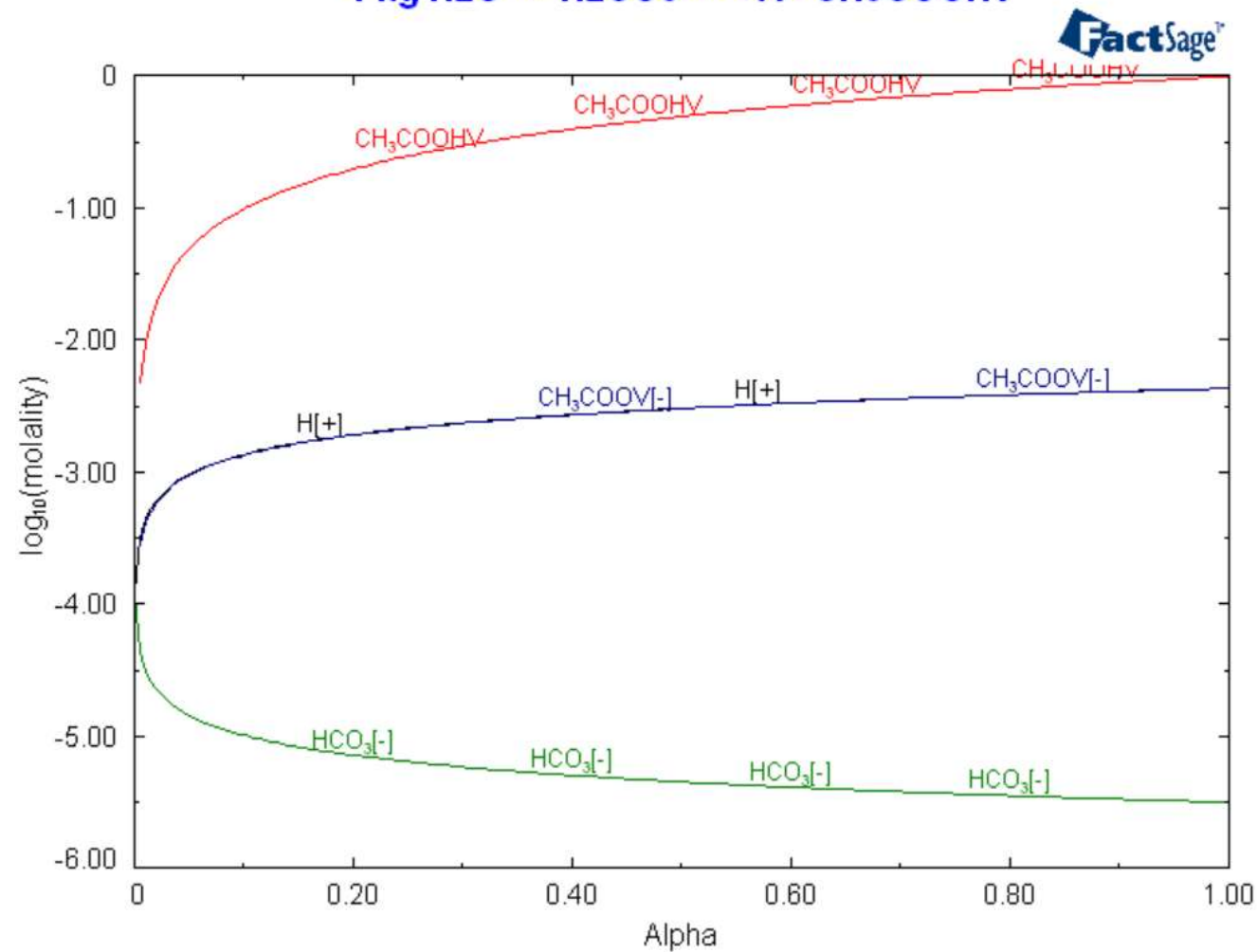
☐ permit selection of 'X' species Help Suppress Duplicates Edit priority list :

Show Selected Select All Select/Clear... Clear OK



Example 1: Water + carbonic acid + acetic acid

1 kg H₂O + H₂CO₃ + <A> CH₃COOHV



Example 2: Anatase → Rutile conversion

[Koukkari, Pajarre, Hack, SGTE Casebook Chapter IV.9]

- The growth of rutile is kinetically hindered and follows the growth law:

$$x_{rutile} = 1 - (1 - kt)^3 \quad \text{with} \quad k = A \exp(-E_A/RT)$$

- The stoichiometric matrix is adjusted as follows:

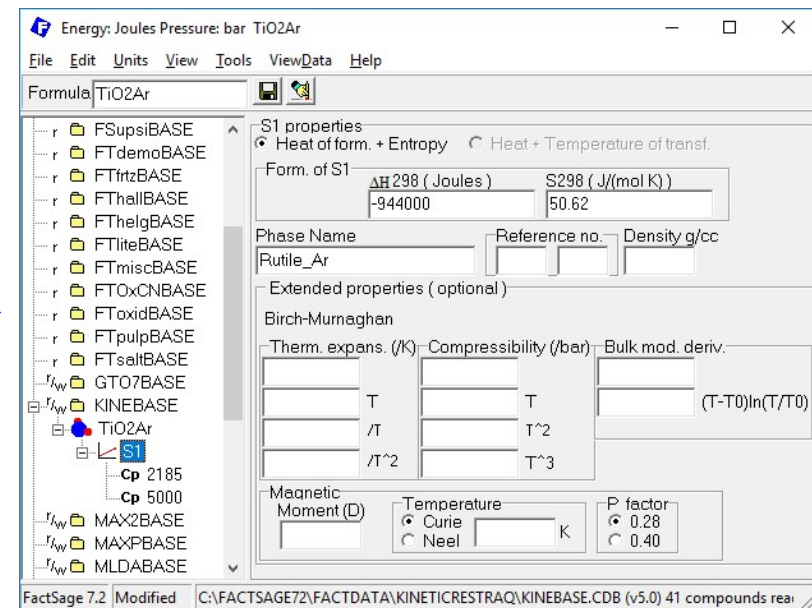
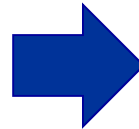
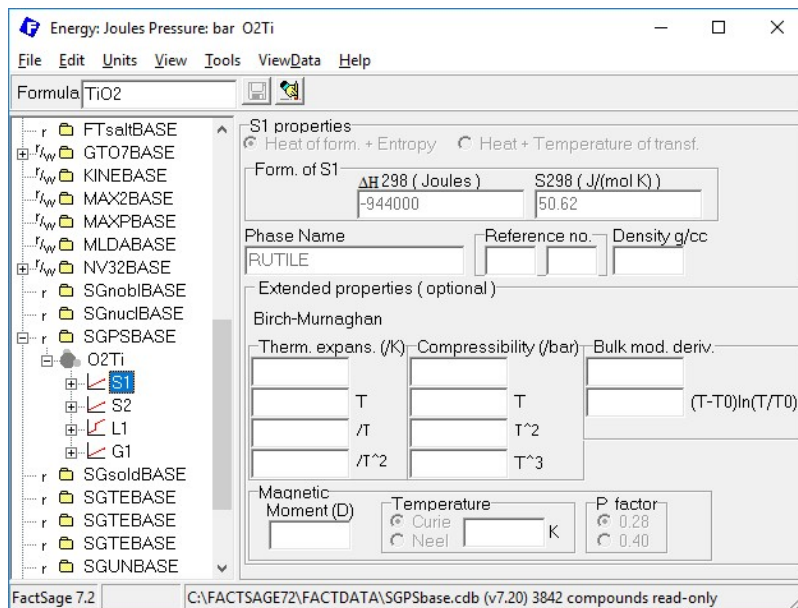
Species	Ti	O	Vi
Rutile	1	2	1
Anatase	1	2	0



Example 2: Anatase → Rutile conversion

[Koukkari, Pajarre, Hack, SGTE Casebook Chapter IV.9]

- In FactSage, **Vi** cannot be entered. Therefore, **Ar** has been used.



Example 2: Anatase → Rutile conversion

[Koukkari, Pajarre, Hack, SGTE Casebook Chapter IV.9]

Selection - Equilib Page 1/1: T(C) = 1000, P(atm) = 1, Alpha = 0.3

File Edit Show Sort

Selected: 2/13 **SOLID**

Page 1/1: T(C) = 1000, P(atm) = 1, Alpha = 0.3

+	Code	Species	Data	Phase	T	V	Activity	Minimum	Maximum
17	T(s)	SGPS	S1		o				
18	T(s2)	SGPS	S2		o				
19	OT(s)	SGPS	ALPHA		o				
20	OT(s2)	SGPS	BETA		o				
21	O2T(s)	SGPS	RUTILE		o				
+	22	O2T(s2)	SGPS	ANATASE	o		1.000	1.000	1.000
	23	O3T(s)	SGPS	S1	o				
	24	O3T(s2)	SGPS	S2	o				
	25	O5T(s)	SGPS	S1	o				
	26	O5T(s2)	SGPS	S2	o				
	27	O7T(s)	SGPS	S1	o				
	28	Ar(s)	KINE	S1	o				
+	29	TiO2Ar(s)	KINE	S1	o		1.000	1.000	1.000

☐ permit selection of "X" species Help Suppress Duplicates Edit priority list:

Show Selected Select All Select/Clear... Clear OK

Equilib - Results 1000 C

Output Edit Show Pages

T(C) P(atm) Energy(J) Mass(mol) Vol(litre)

FactSage 7.2

T = 1000 C
P = 1 atm
V = 0 dm3

STREAM CONSTITUENTS

CONSTITUENT	AMOUNT/mol
TiO2	1.0000E+00
Ar	3.0000E-01

EQUIL AMOUNT

CONSTITUENT	AMOUNT/mol	ACTIVITY
O2Ti_ANATASE(s2)	7.0000E-01	1.0000E+00
TiO2Ar_S1(s)	3.0000E-01	1.0000E+00

Cp	H	S	G	V
J.K-1	J	J.K-1	J	dm3

7.50606E+01	-8.68054E+05	1.48881E+02	-1.05760E+06	0.00000E+00

Cp	H	S	G
J.K-1	J	J.K-1	J

5.17787E+01	-6.05257E+05	1.04366E+02	-7.38131E+05
2.32819E+01	-2.62796E+05	4.45147E+01	-3.19470E+05

Cut-off limit for phase activities = 1.00E-70

Databases: KINE 7.0, SGPS 7.2

Data Search options: exclude gas ions; organic CxHy.. X(max) = 2; min soln cpts = 2



Immaterial system components

System	Constraint	Matrix element	Conjugate potential	Practical examples
Chemical equilibrium systems	$\sum_{k=1}^N a_{kj} n_k = b_j$	a_{kj}	$\pi_j = \frac{\partial G}{\partial b_j}$	Multiphase chemical equilibrium Phase diagrams
Systems with area constraints	$\sum_{k=1}^{N_s} A_k n_k^s = A$	$\frac{A_k}{A_0}$	$\pi_{area} = \frac{\partial G}{\partial b_{(area)}} = \sigma A_0$	Surface and interface systems, sorption phenomena
Systems with volume constraints	$\sum_{k=1}^{N_F} V_k n_k^F = V_F$	$\frac{V_k}{V_0}$	$\pi_F = \frac{\partial G}{\partial b_{(V_F)}} = \Pi \bar{V}_F$	Osmotic systems, fibers and membranes
EOR-controlled complex systems	$\sum_{k=1}^N a_{kr} n_k = b_{(r)}$	v_{kr}	$\pi_{(r)} = \frac{\partial G}{\partial b_{(r)}} = \frac{\partial G}{\partial z_i^r} = -Aff_i$	Reactive systems Partial and 'super' equilibria

Para-equilibrium systems	$u_k = \frac{n_k}{\sum_M n_M} = const.$	$\frac{n_{Fe}}{n_M} a_{k,M} - a_{k,Fe}$	$\frac{\partial G}{\partial z} = \sum_{M^*} a_{k,M^*} \left(\frac{\partial G}{\partial b_{M^*}} \right) = \sum_{M^*} a_{k,M^*} \pi_{M^*}$	Paraequilibria, co-precipitation
System with external magnetic field	$B = const.$	$\frac{\Delta_M \mu_i}{f(B) M_0}$	$\pi_M \equiv \frac{\partial G}{\partial b_M} = f(B) M_0$	Magnetic phase stability
Electrochemical (Donnan) multi-phase systems	$\sum_{k=1}^{N_a} z_k n_k^a = Q^a$	z_k	$\pi_{(q^a)} = \frac{\partial G}{\partial b_{(q^a)}} = F \Delta \phi^a$	Electrochemical membrane systems, Pulp suspensions
Constant contribution pH	$pH = const.$	$N_{H,k}$	$\pi_{Ha} \equiv RT \ln 10 pH_a$	Biochemical systems (pH = 7 at standard state)
Constant contribution ionic strength	$I = \frac{1}{2} \sum_k z_k^2 c_k = const.$	z_k^2	$\pi_I \equiv \frac{-\alpha RT \sqrt{I}}{1 + B \sqrt{I}} = \frac{RT}{z_k^2} \ln \gamma_k$	Biochemical systems with constant ionic strength

Pertti Koukkari, *Introduction to constrained Gibbs energy methods in process and materials research*

www.vtt.fi/inf/pdf/technology/2014/T160.pdf



Grain boundary partitioning

- Grain boundary density can be considered as constraint.
- Thus, grain boundaries can be treated as another phase associated with a virtual systems component.



Grain boundary partitioning

ARTICLE OPEN

A machine learning approach to model solute grain boundary segregation

Liam Huber¹, Raheleh Hadian¹, Blazej Grabowski¹ and Jörg Neugebauer¹

Even minute amounts of one solute atom per one million bulk atoms may give rise to qualitative changes in the mechanical response and fracture resistance of modern structural materials. These changes are commonly related to enrichment by several orders of magnitude of the solutes at structural defects in the host lattice. The underlying concept—segregation—is thus fundamental in materials science. To include it in modern strategies of materials design, accurate and realistic computational modelling tools are necessary. However, the enormous number of defect configurations as well as sites solutes can occupy requires models which rely on severe approximations. In the present study we combine a high-throughput study containing more than 1 million data points with machine learning to derive a computationally highly efficient framework which opens the opportunity to model this important mechanism on a routine basis.

npj Computational Materials (2018)4:64; doi:10.1038/s41524-018-0122-7

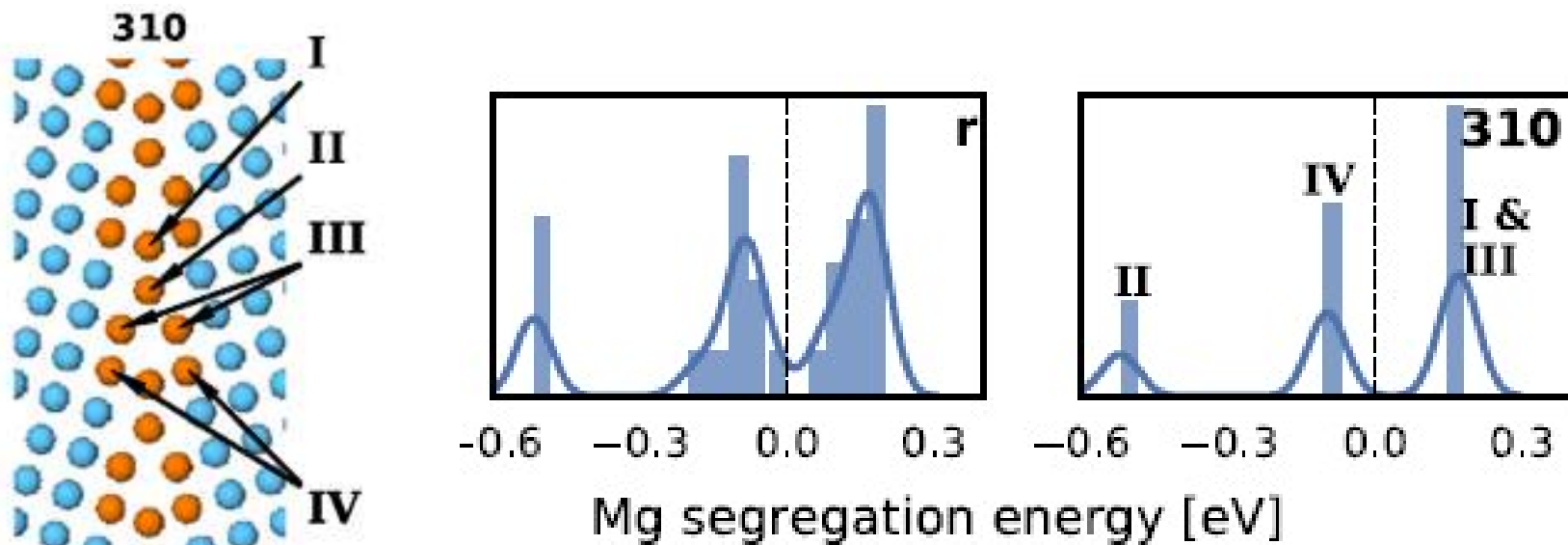


Grain boundary partitioning

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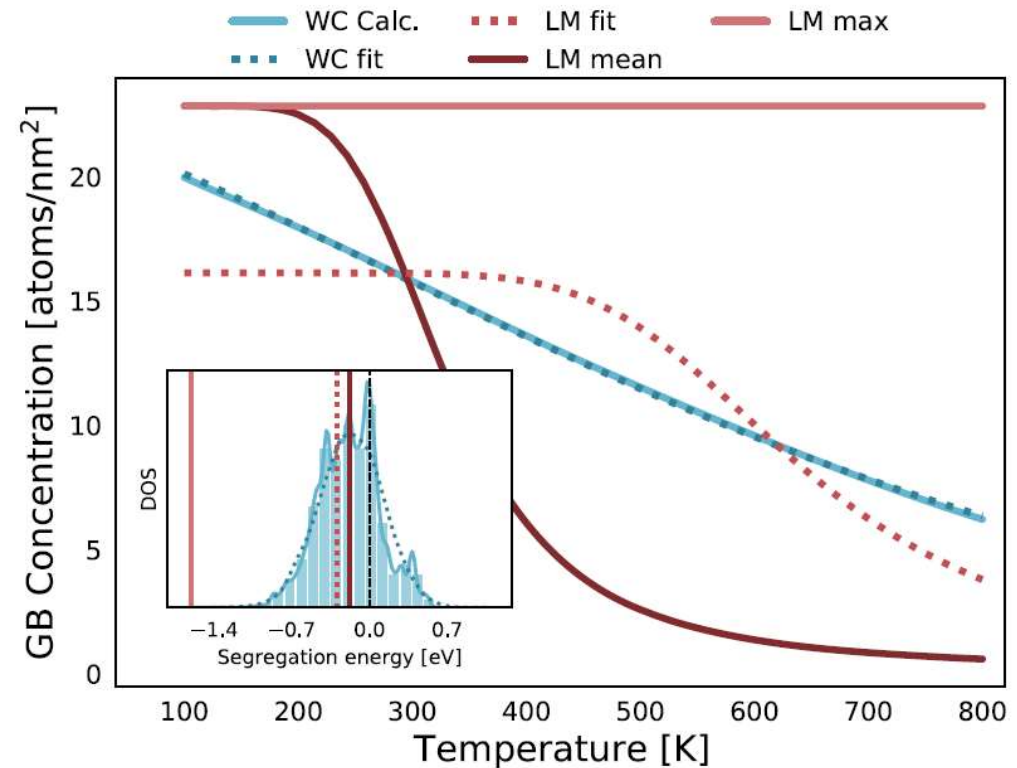
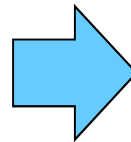
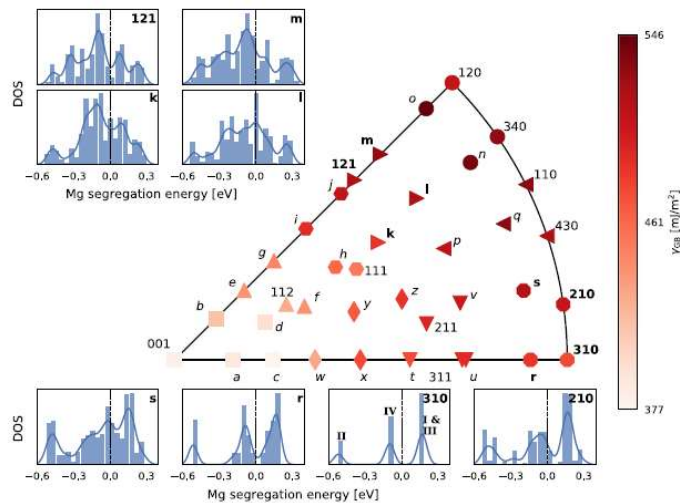


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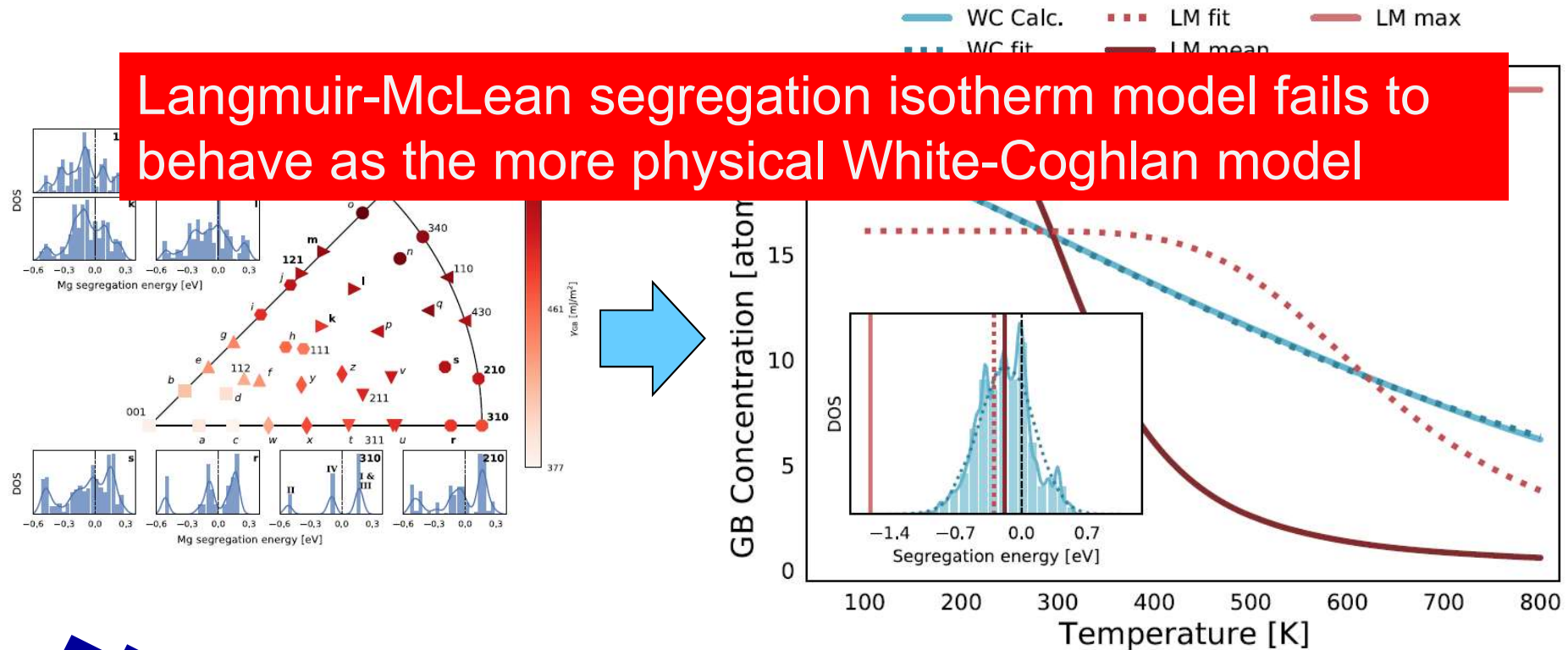


Grain boundary partitioning

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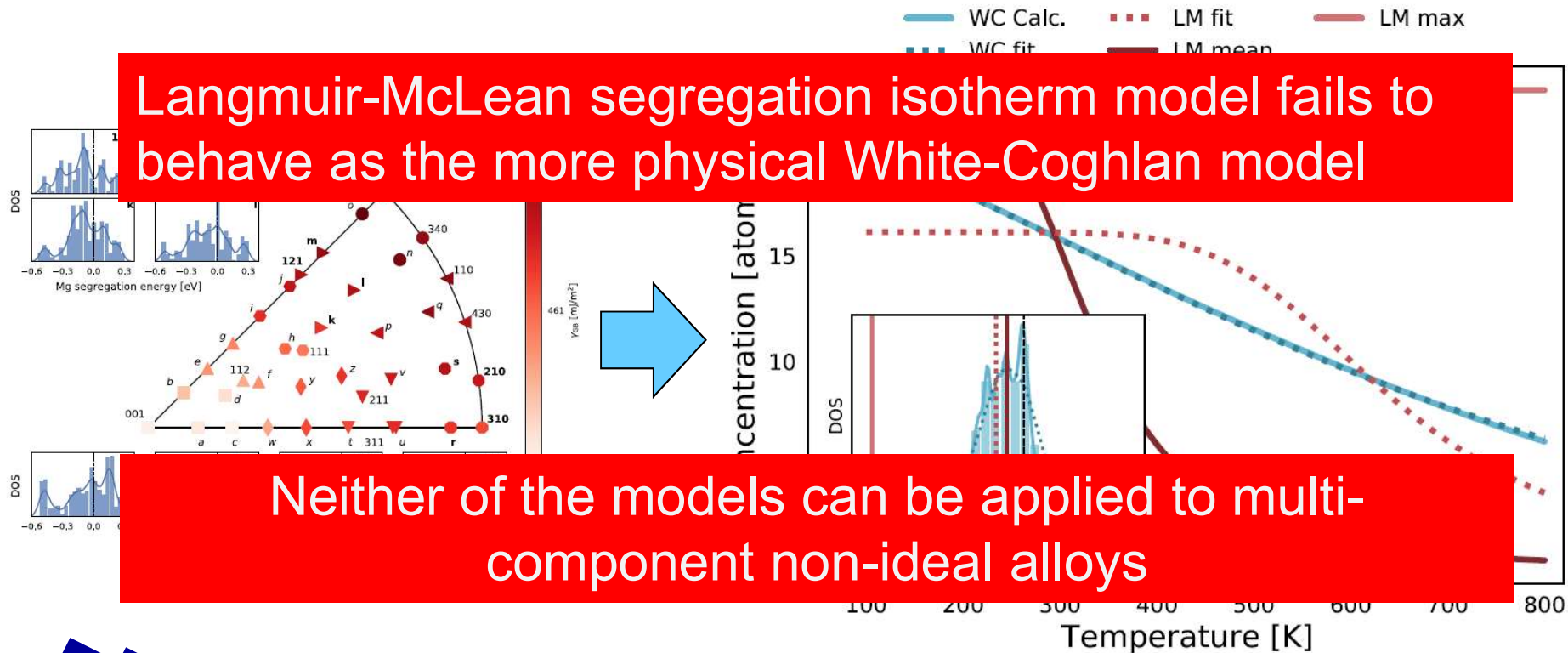


Grain boundary partitioning

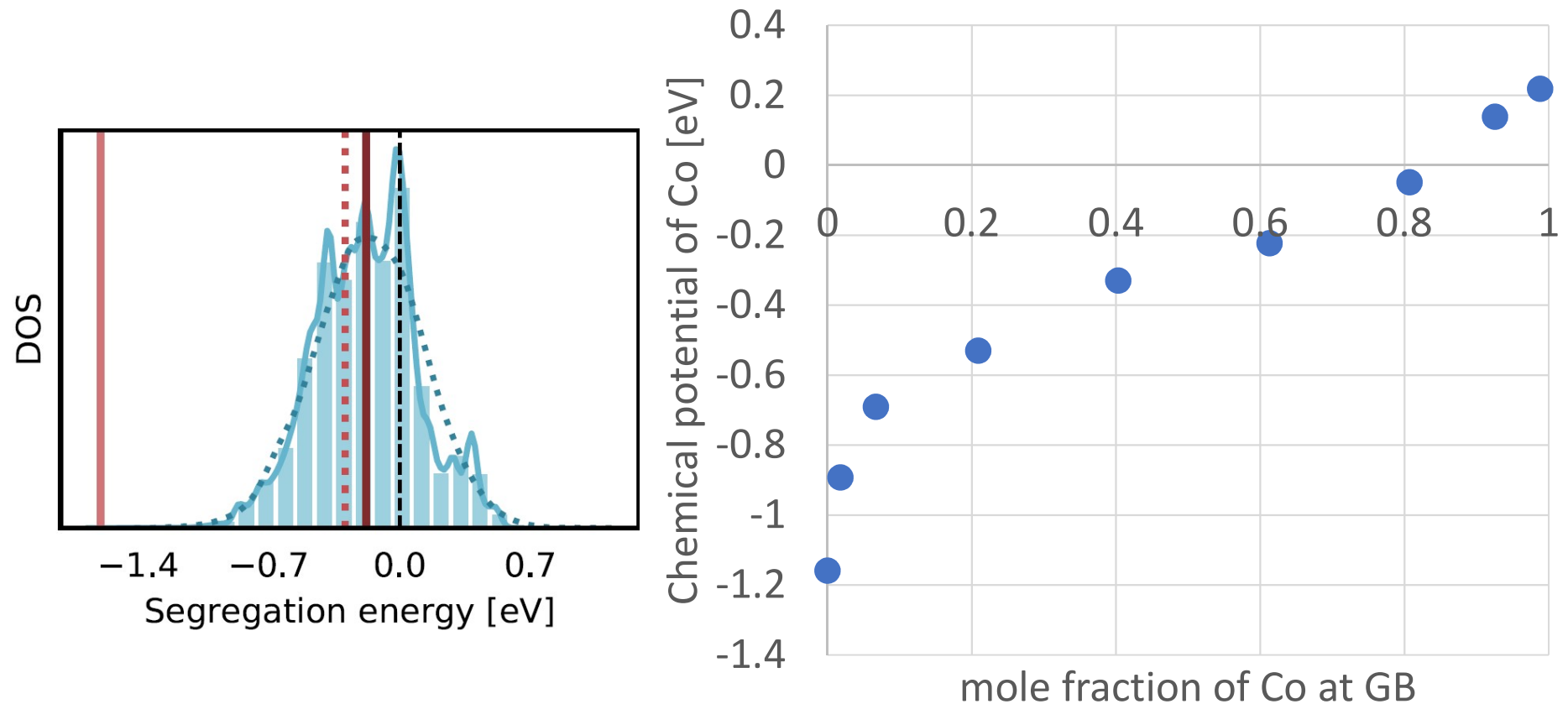
ARTICLE OPEN

A machine learning approach to model solute grain boundary segregation

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Grain boundary partitioning



Grain boundary partitioning

FactSage 7.3: Solution

File Edit Units Options Tools Help

Function Name ACGBsoln.FCGB

Solution Name	Created	Last Modified	P factor
FCGB	2019.06.04	2019.06.04	☒ 0.28 ☐ 0.40

Model Name Compound Energy Formalism Lattices 2

Solution description ...

GrainBoundary
Original ChemSage file: C:\FactSage73\ChemSage\SGTE-AlCo.dat

Solutions (2)

- FCC_ (12-2) (SUBL)
 - SubLattices
 - A (2)
 - Al
 - Co
 - B (1)
 - Va (A)
 - End Members (2)
 - (0) Al:Va
 - (1) Co:Va
 - Mixables (0)
 - Interactions (7)
- FCGB (12-2) (SUBL)
 - SubLattices
 - A (2)
 - AlGB
 - CoGB
 - B (1)
 - Gd (A)
 - End Members (2)
 - (0) AlGB
 - (1) CoGB
 - Mixables (0)
 - Interactions (7)



Grain boundary partitioning

Equilib - Reactants

File Edit Table Units Data Search Data Evaluation Help

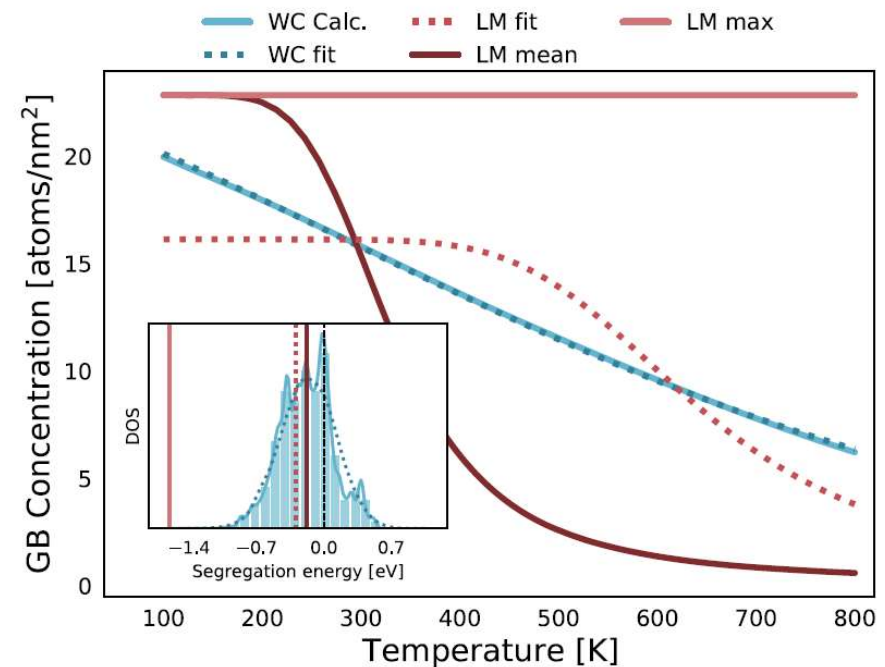
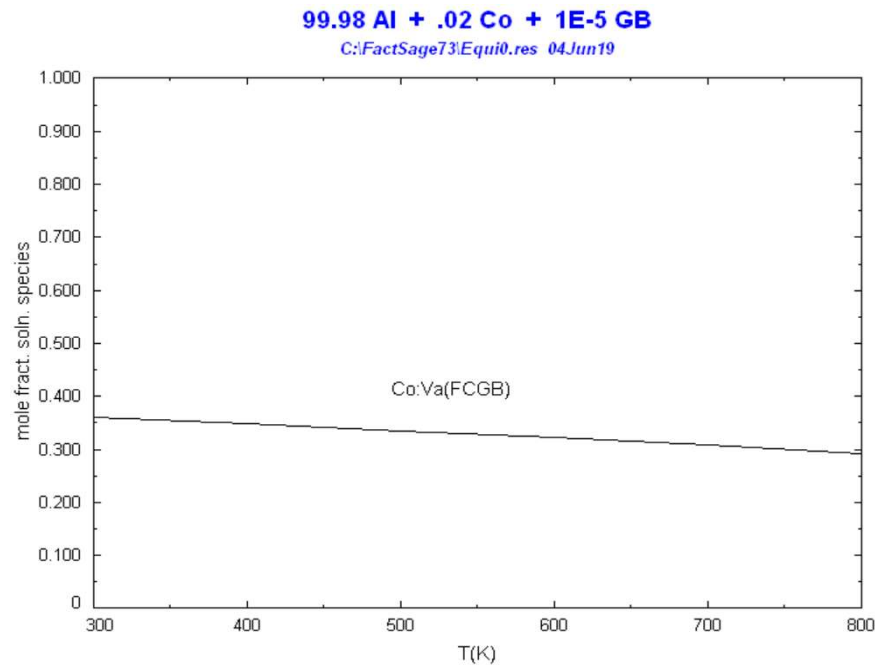
T(K) P(atm) Energy(J) Quantity(mol) Vol(litre)

1 - 3

	Quantity(mol)	Species	Phase	T(K)	P(total)**	Stream#	Data
	99.98	Al				1	
+	.02	Co				1	
+	1E-5	Gd				1	



Grain boundary partitioning



The behavior of the non-optimized FactSage model resembles the WC-model.
The FactSage model can be applied to multi-component non-ideal alloys.



Conclusion

- There are many ways how constraints can be considered semi-empirically in thermodynamic calculations.
- New routes and ideas will emerge!



Thank you for your attention!

